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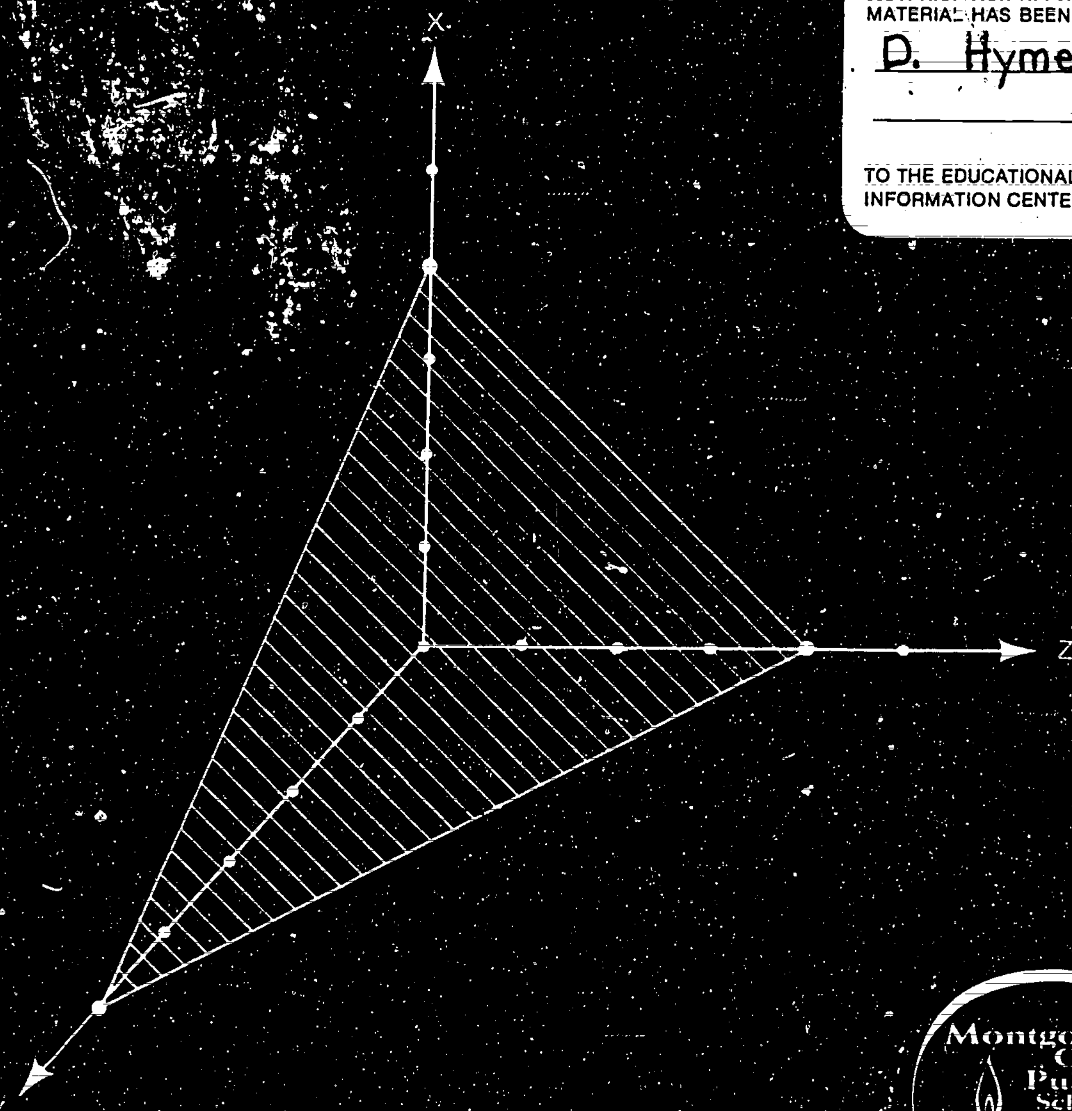
ABSTRACT

This guide was prepared as an instructional aid for teachers of a second-year course in algebra. It was designed to be applicable to a wide range of Algebra II programs offered in senior high schools. The content of the Algebra II program has been divided into 13 major units and a preliminary unit designed to review selected Algebra I skills. Each of the 13 major units include the following features: the purpose for including the unit in the Algebra II program: an overview of the section: a vocabulary list: a list of performance objectives: suggestions to the teacher, including the number of instructional days per unit: four assessment tasks for each performance objective: textbook cross references: and an answer key to the assessment tasks. Several sections also include entering performance objectives, a diagnostic test keyed to entering performance objectives, and a diagnostic test answer key. (MP)

ALGEBRA II INSTRUCTIONAL GUIDE



Montgomery County Public Schools

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POINT OF VIEW FOR MATHEMATICS EDUCATION IN MONTGOMERY COUNTY

A provocative activity which teachers often use with pupils at various levels is that of trying to imagine what a world without numbers would be like. Such a world is difficult to imagine. The idea of number continues to play an important role in virtually all aspects of our world; and mathematics, therefore, constitutes a program of considerable importance in the schools.

As a discipline, mathematics is truly the art and science of abstraction. Characteristics of the physical world are converted into abstract ideas and symbols; these are then manipulated through mathematical operations to produce information and theorems about less easily observed aspects of the world. Recent evidence supports the contention that children's experiences with concrete materials are vital to later conceptual development. The school program thus proceeds from the concrete to the abstract.

The concepts of mathematics acquire greater meaning when they can be applied to the world in which we live. Because the variety and extent of mathematical applications have grown so rapidly in recent years, it is impossible for any one person to be conversant with the entire field. The school program must therefore be developed so that mathematical applications are selected and presented as efficiently as possible and with the intent of challenging pupils at all levels to see mathematics as an independent discipline as well as a tool for the advancement of other disciplines.

The Montgomery County mathematics program is designed and implemented to take into account the logical and relatively sequential nature of mathematics. Equally important, too, is the realization that the rate at which individual students learn mathematics varies significantly. The mathematics program is structured to encourage various approaches which allow students to progress at their individual rates.

The pre-algebra objectives range over six areas of mathematics and are arranged according to 14 different levels of achievement. Assessment measures have been constructed for each objective so that individual progress can be measured in a variety of categories. Enrichment activities are available for both the able student and the student who needs reinforcement.

Several options are available to the student who has completed the pre-algebra objectives. Differentiated paths through a variety of courses are available to the student, as can be seen in the Mathematics Program Patterns Chart, on page xi Each student has available a sequence of courses which can be suited to his/her interests and abilities.

Computer literacy is addressed at several levels of the mathematics program; career information is incorporated as appropriate throughout. Consumer applications are taught as mathematical skills are developed; the mathematics of consumerism is further emphasized in an elective senior high course.

In general terms, the instructional program in mathematics should help each student to:

Develop basic skills in using the vocabulary and symbols of mathematics

Develop skills in recognizing common geometric shapes

Develop basic skills in computing

Develop basic skills in working with geometric shapes

Develop basic skills in measuring, graphing, and using tables and charts

Develop understanding of the vocabulary and symbols of mathematics

Develop understandings necessary for translating among mathematical symbols, words, and the physical world

Develop concepts related to common geometric shapes

Develop understanding of computation

Develop understanding of measurement

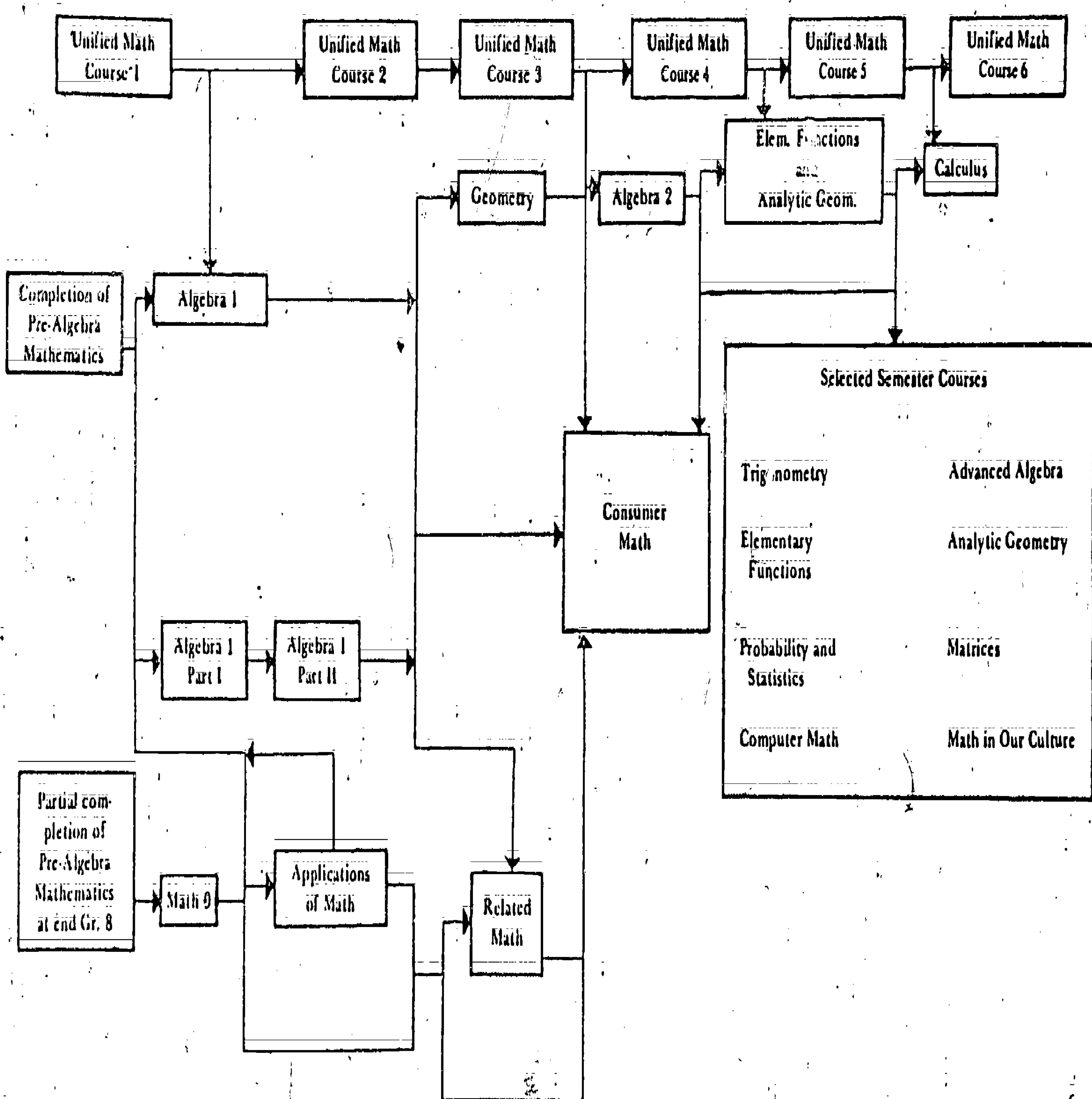
Develop an understanding of basic principles related to the structure of mathematics

Develop understanding and basic skills in problem solving

Apply the principles of mathematical reasoning to the solution of problems

Appreciate the significance of mathematics in daily living and its contribution to our cultural heritage

Use mathematics as needed in daily living



MATHEMATICS PROGRAM PATTERNS CHART

RECOMMENDED ALGEBRA 2 TIME ALLOCATION

UNIT	NUMBER OF DAYS
Preliminary (Review)	10
I	10
II	10
III	12
IV	20
V	10
VI	15
VII	10
VIII	20
IX	15
X	12
XI	20
XII	10

The time allocations include both testing and instructional days. Three extra days are allowed in each semester to provide for flexibility in planning.

It is the practice in many schools to enable accelerated tenth grade students to master topics from trigonometry along with those of Algebra 2. When this is the case, the Algebra 2 instruction should be condensed into 145 days, allowing a minimum of 35 days for the unit on trigonometry.

INTRODUCTION

ORGANIZATION OF UNITS IN THE COURSE

The order of units in this instructional guide provides a logical scheme for incorporating the real and complex number systems throughout Algebra 2. The order of the units does not conform necessarily to the order of topics found in the approved texts. The cross-reference key was included to assist teachers with planning. With the exception of the complex numbers, the first six units present the continued and extended growth of elementary algebra topics. The remaining sections are an ordered development of relations and functions.

ORGANIZATION OF INDIVIDUAL UNITS

Certain units are preceded by "entering performance objectives." A diagnostic test keyed to these objectives determines the proficiency level of the skills needed for the study of the unit.

Each unit contains the performance objectives, a cross-referenced guide to currently approved Algebra 2 textbooks, and a vocabulary list. At least four sample assessments are provided for each performance objective. An answer key for the sample assessment items concludes the unit. Alternative answers and procedures may occur, although not all acceptable answers are listed. When no other specific instructions are given, the student is expected to give all answers in simplest form.

REVIEW OF SELECTED ALGEBRA 1 SKILLS

The guide provides a preliminary section designed to diagnose the level of the student's basic Algebra 1 skills. The allocated time for this review will vary with individual situations; however, review time should not exceed ten days.

SUGGESTIONS TO THE TEACHER

"Suggestions to the Teacher" assist in the implementation of the performance objectives. The topics include identification of problem areas; use of instructional aids and references; application of calculators and computers; and the time allocation for the unit. The higher order assessment measures and enrichment objectives are included for the more able students. Career applications and activities found in approved textbooks are also listed. Ten broad career categories were selected from the 979 occupations listed in the Dictionary of Occupational Titles (U.S. Department of Labor).

INTENT

Algebra 2 not only completes the complex number system by incorporating real and imaginary numbers but also extends the skills of problem solving. The course provides a continual review of basic algebraic skills as well as preparation for further studies in the sciences.

RATIONALE FOR THE ALGEBRA 2 INSTRUCTIONAL GUIDE

The Algebra 2 Instructional Guide has been constructed to assist the teacher in planning a program which will facilitate student attainment of the Algebra 2 objectives as described on pages 7 and 8 of the "Mathematics 9-12" section of the Program of Studies of the Montgomery County Public Schools (1979). A listing of the Algebra 2 instructional guide objectives from the Program of Studies follows, with the appropriate unit from this instructional guide noted:

Instructional Objectives	Unit from Guide
Upon completion of the course, the student should be able to:	
• Perform the four fundamental operations with irrational numbers	I
• Perform the four fundamental operations with complex numbers	II
• Factor or simplify rational expressions	III
• Distinguish between a function and a relation which is not a function	IV
• Identify and graph the equations of functions which are constant, linear, or quadratic	IV
• Use the relationship of the slopes of parallel or perpendicular lines to determine equations of such lines	V
• Solve direct variation problems	V
• Solve quadratic equations over the set of reals or over the set of complex numbers	VI
• Solve quadratic inequalities over the set of real numbers and sketch the graph of the inequality	VI
• Apply synthetic substitution, the Remainder Theorem, Factor Theorem, Rational Root Theorem, Fundamental Theorem of Algebra and the Property of Continuity to the estimation of zeros of polynomials	VII
• Identify the graph of a quadratic relation as a circle, ellipse, hyperbola, or other	VIII
• Sketch the graphs of the circle, the ellipse, the parabola, and the hyperbola from the quadratic equation which describes each	VIII
• Solve problems involving combined variations	VIII

- Solve problems involving systems of linear equations in two or three variables IX
- Sketch graphs of and find solutions for quadratic-linear and quadratic-quadratic systems of equations IX
- Multiply, divide, or simplify expressions containing real number exponents X
- Solve equations containing radical expressions X
- State equivalent exponential and logarithmic equations XI
- Apply scientific notation, interpolation, and common logarithms to compute products, quotients, and powers XI

Units XII and XIII of the guide provide materials for acceleration and enrichment which can be used for students ready for them.

APPROVED TEXTS

Dolciani, Mary P., et al. Modern Algebra and Trigonometry - Structure and Method. (Revised edition). Boston: Houghton Mifflin Company, 1973.

---. Algebra 2 and Trigonometry. Boston: Houghton Mifflin Company, 1978.

---. Algebra 2 and Trigonometry. (Revised edition). Boston: Houghton Mifflin Company, 1980.

Foster, Alan G., et al. Algebra Two with Trigonometry. Columbus Charles E. Merrill Publishing Company, 1979.

Keedy, Mervin L., et al. Algebra Two. Menlo Park, Ca.: Addison Wesley Publishing Company, 1978.

Payne, Joseph N., et al. Algebra Two with Trigonometry. New York: Harcourt Brace Jovanovich, Inc., 1977.

Sobel, Max A., and Banks, J. Houston. Algebra: Its Elements and Structure, Book 2 (Third edition). New York: Webster Division, McGraw-Hill Book Company, 1977.

Sorgenfrey, Robert H., et al. Modern Algebra and Trigonometry, Structure and Method, Book 2 (New edition). Boston: Houghton Mifflin Co., 1973.

Travers, Kenneth J. Using Advanced Algebra. River Forest, Ill.: Laidlaw Brothers, 1973.

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Coan, James S. BASIC BASIC - An Introduction to Computer Programming in Basic Language. Rochelle Park, N.J.: Hayden Book Company, Inc., 1970. (Approved for MCPS course Computer Mathematics.)

Golden, Neal. Computer Programming in the BASIC Language. New York: Harcourt Brace Jovanovich, 1975. (Approved for MCPS course Computer Mathematics)

Johnson, Richard E., et al. Algebra and Trigonometry (Second edition). Menlo Park, Ca.: Addison Wesley Company, 1971. (Formerly an approved text in MCPS.)

Vogeli, Bruce R., et al. Algebra Two and Trigonometry. Morristown, N.J.: Silver Burdett Company, 1976.

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OUTLINE OF COURSE CONTENT

BASIC UNITS

Preliminary	A Review of Selected Algebra 1 Skills
I.	Irrational Numbers
II.	Complex Numbers
III.	Factoring and Simplifying Rational Expressions
IV.	Functions and Relations
V.	Points and First-Degree Equations
VI.	Solving Second Degree Equations in One Variable
VII.	Solving Equations of Higher Degree
VIII.	Conic Sections
IX.	Systems of Open Sentences
X.	Real Number Exponents
XI.	Exponential and Logarithmic Functions
XII.	Trigonometry
XIII.	Sequences, Series, and the Binomial Theorem

PRELIMINARY UNIT - A REVIEW OF SELECTED ALGEBRA 1 SKILLS

PURPOSE

The purpose of this preliminary unit is to provide both teachers and students with evaluative data on the Algebra I skills of beginning students of Algebra II.

The ten performance objectives listed later in this unit define minimal algebra skills that -- in the opinion of the authors -- are required throughout the study of Algebra 2.

There are, certainly, other algebra skills needed by the Algebra 2 student which have not been included in the preliminary unit. These entering performance objectives will be handled in later units, as described in the introduction to this guide.

SUGGESTIONS TO THE TEACHER

The teacher would find it advantageous to open the course in Algebra 2 by administering the diagnostic test which follows the CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS. The items of this test are keyed to the statement of each unit performance objective; each performance objective is keyed to currently approved Algebra 2 textbooks. The test should take approximately 90 minutes.

In the event that a student demonstrates severely deficient performance on the diagnostic assessment items of this preliminary unit, program alternatives should be provided. In some cases, the teacher should advise students to devote time to a long-term, algebra-skills remediation program. In other cases, students should be advised to elect different mathematics courses; e.g. Consumer Mathematics or Related Mathematics.

The results of this test can provide the bases from which initial instructional activities may proceed. Suggested activities include: short teacher presentations on certain topics that need added emphasis; supplementary sets of exercises keyed to specific student needs; student activities in small groups; skills labs, including the SRA Algebra Skills Kit and the SRA Computation Skills Kit. The teacher may find it helpful to refer to recent Algebra 1 textbooks for additional instructional suggestions and exercise ideas. The Montgomery County Public Schools Algebra 1 Instructional Guide represents a comprehensive resource on the subject.

The recommended maximum number of instructional days for this unit is ten. Teachers are reminded that Unit Performance Objective 8 expects the student to raise a real number to a positive integral power. It would be helpful to students, at later stages of the course, to have facility with this process.

PRELIMINARY UNIT - A REVIEW OF SELECTED ALGEBRA 1 SKILLS

PERFORMANCE OBJECTIVES

1. Perform indicated operations with signed numbers.
2. Identify real number axioms.
3. Identify and apply set notation.
4. Identify various subsets of the real numbers.
5. Apply the definition of absolute value.
6. Determine the solution set of a linear equation in one variable.
7. Determine the solution set of a linear inequality in one variable. Express the solution set in correct set notation.
8. Evaluate a formula by the substitution principle.
9. Determine the value of an expression, using the order of operations rule.
10. Translate a narrative statement into an open sentence.

PRELIMINARY UNIT

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)	Algebra I Instructional Guide Objectives
1	22-30	13-25	11-25	--	7-19	A8-A17	5-6	44-52	5-10	Unit III 5, 7, 8, 10, 11
2	8-17	8-25	6-25	6-9	11-22	A1-A7	7-10	16-23	18-20	Unit III A: 2 B: 2, 6, 9
3	1-8	2-4	1-3	--	--	A1-A7 A9-A17	57-63	5-8	15	Unit I 1, 3, 5
4	--	4-5	4-5	45	2-5	A1-A7 2-5	16-20 48-51	1-2	2 4	Unit I: 2 Unit III: 3
5	14-18	60-63	58-60	18-20	40-42	A1-A7 7 112-114	63-66	39-43	5	Unit III 4, 12
6	44-47	37-40	35-37	10-13	48-56	102-104	36	54-59	14-15 34-36	Unit IV 2 - 6
7	48-52	49-55	49-52	21-24	130-142	A9-A17 115-118 127-128	58-59 63-66	59-64	42-48	Unit IV 7 - 11
8	47-48	9-10	--	2-4	21	A8-A17 108-111	12 31	58-59	37	Unit II 4
9	29-30	24-25	23-25	2-4	20-22	--	--	21 51-56	11-13	Unit II 2,3
10	60-67	42-48	38-44	16 27-28	57-58	121 123	35-37	68-74	13 16	Unit IV 12-15

PRELIMINARY UNIT - A REVIEW OF SELECTED ALGEBRA 1 SKILLS

DIAGNOSTIC TEST KEYED TO PERFORMANCE OBJECTIVES

Directions: Read each section carefully and supply the requested information. Express answers in simplest form.

1. Perform the indicated operations.

<u>Integers</u>

a) $(-16) + (^+10) =$? a) ______

b) $(-63) + (-11) =$? b) ______

c) $(-6) - (^+6) =$? c) ______

d) $(12) - (-15) =$? d) ______

e) $(-5)\ (^+6) =$? e) ______

f) $(-7)\ (-8) =$? f) ______

g) $(-20) \div (-5) =$? g) ______

h) $(-250) \div (+125) =$? h) ______

i) $(+6) + (-7) + (-9) + (+4) =$? i) ______

<u>Fractions</u>

j) $(-\frac{3}{8}) + (-\frac{5}{8}) =$? j) ______

k) $-\frac{3}{8} - \frac{7}{12} =$? k) ______

l) $\frac{2}{7} \times \frac{3}{8} =$? l) ______

m) $-\frac{3}{4} \div \frac{5}{2} =$? m) ______

n) $\frac{5}{16} \times -(1\frac{9}{15}) \times 3\frac{1}{8} =$? n) ______

o) $\frac{3}{8} : 2\frac{1}{4} =$? o) ______

<u>Decimals</u>

p) $2.05 + 7.3$ p) ______

q) $(-0.637) - (-1.2)$ q) ______

r) $(.001) \times (-.32)$ r) ______

s) $(7.5) \div (.025)$ s) ______

2. Fill in the blanks as directed.

a) Listed below are several statements of equality. Select the axiom of real numbers illustrated by each equality.

Axioms of real numbers

(1) commutative property of addition

(2) associative property of addition

(3) commutative property of multiplication

(4) associative property of multiplication

(5) distributive property

Statements of equality

(1) $7 \cdot 2 = 2 \cdot 7$ (1) ____________

(2) $6 + (3 + 5) = (6 + 3) + 5$ (2) ____________

(3) $9 + (2 \cdot 3) = (2 \cdot 3) + 9$ (3) ____________

(4) $3(6 + 1) = 3 \cdot 6 + 3 \cdot 1$ (4) ____________

(5) $4 \cdot (3 \cdot n) = (4 \cdot 3) \cdot n$ (5) ____________

b) Complete the following statements

(1) The multiplicative inverse of $\frac{3}{7}$ is ? (1) ____________

(2) The multiplicative inverse of -9 is ? (2) ____________

(3) The additive inverse of $\frac{5}{7}$ is ? (3) ____________

(4) The additive inverse of 6 is ? (4) ____________

3. Fill in the blanks as directed.

a) Specify, by a roster, the set of odd numbers between 1 and 10:

a) ____________

b) Write the number of the choice that represents the null or empty set:

b) ____________

(1) 0.

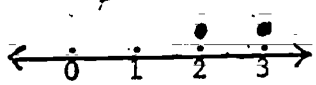
(2) $\{0\}$.

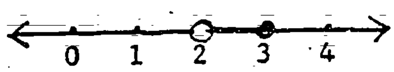
(3) $\emptyset$.

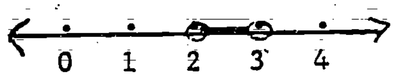
(4) $\{\emptyset\}$.

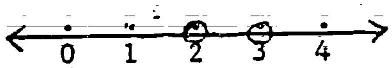
c) Select the graph that represents the set $\{x: 2 < x < 3\}$ from the following:

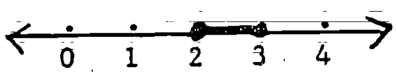
c) ____________

(1) 

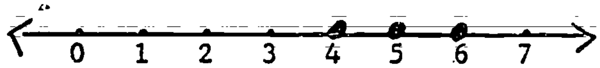
(2) 

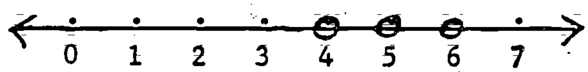
(3) 

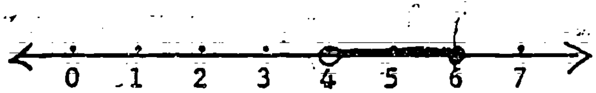
(4) 

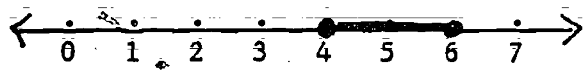
(5) 

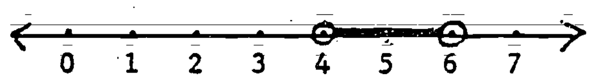
d) Select the graph that represents the set $\{x: 4 \leq x \leq 6\}$ from the following:

(1) 

(2) 

(3) 

(4) 

(5) 

4. Listed below are illustrations of sets. Match each illustration with its corresponding name.

(1) real numbers

(2) natural numbers

(3) whole numbers

(4) integers

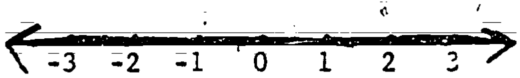
(5) rational numbers

a) $\{0, 1, 2, 3, \ldots\}$ a) ____________

b) $\{\ldots, -3, -2, -1, 0, 1, \ldots\}$ b) ____________

c) $\{1, 2, 3, \ldots\}$ c) ____________

d) $\{x : x = \frac{a}{b}$, where a and b are in integers, $b \neq 0\}$ d) ____________

e) 

e) ____________

5. Apply the definition of absolute value to evaluate each of the following:

a) $|-8| = ?$ a) ____________

b) $-|-8| = ?$ b) ____________

c) $-|x| = ?$ c) ____________

6. Solve each of the following open sentences:

a) $x + \frac{1}{2} = -4$ a) ____________

b) $5 - y = -10$ b) ____________

c) $-15m = 52$ c) ____________

d) $\frac{x}{4} = -3$ d) ____________

e) $-6(6 + 4x) = 12$ e) ____________

f) $9 + 2(x + 1) = 19$ f) ____________

g) $9x - 2 = 4(x + 7)$ g) ____________

7. Solve each of the following open sentences. Express the solution set in correct set notation.

a) $-6y \geq 13$ a) ____________

b) $5(2a - 1) \leq -15$ b) ____________

c) $6 + |3x + 4| \leq 8$ c) ____________

8. In each exercise below, solve for the <u>underlined</u> variable by using the numerical values provided. Do <u>not</u> assign a value for π.

a) $\underline{A} = \pi r^2$ when $r = 4$ a) ____________

b) $F = \frac{9}{5}\underline{C} + 32$ when $F = -4$ b) ____________

c) $A = \frac{1}{2}h(\underline{b_1} + b_2)$ when
$A = 20,\ h = 4,\ b_2 = 3$ c) ____________

d) $\underline{V} = \frac{4}{3}\pi r^3$ when $r = 3$ d) ____________

9. Determine the value of an expression using the order of operations rules.

a) $8 \div 4 + 6 - 7 \times 4$ a) ______________

b) $\frac{2 [3x + 4(2x + 3)]}{3(2 + 3^2)}$ b) ______________

10. Translate the verbal statement into an open sentence. (Do not solve.)

The sum of the squares of two consecutive integers is greater than 25.

____

ANSWERS

1. a) -6
   b) -74
   c) -12
   d) 27
   e) -30
   f) 56
   g) 4
   h) -2
   i) -6
   j) -1
   k) $-\frac{23}{24}$
   l) $\frac{3}{28}$
   m) $-\frac{3}{10}$
   n) $-\frac{25}{16}$ or $-1\frac{9}{16}$
   o) $\frac{1}{6}$
   p) 9.35
   q) 0.563
   r) - .00032
   s) 300

2. a)
   (1) 3
   (2) 2
   (3) 1
   (4) 5
   (5) 4

2. b) (1) $\frac{7}{3}$
   (2) $-\frac{1}{9}$
   (3) $-\frac{5}{7}$
   (4) - 6

3. a) $\{3, 5, 7, 9\}$
   b) 3
   c) 3
   d) 4

4. a) 3
   b) 4
   c) 2
   d) 5
   e) 1

5. a) 8
   b) -8
   c) x, if $x < 0$
   -x, if $x > 0$

ANSWERS

6. a) $-\frac{9}{2}$ or $-4\frac{1}{2}$

   b) 15

   c) $\frac{-52}{15}$ or $-3\frac{7}{15}$

   d) -12

   e) -2

   f) 4

   g) 6

7. a) $\{ y : y \le -\frac{13}{6} \}$

   b) $\{ a : a \le -1 \}$

   c) $\{x : x \le -\frac{2}{3} \cup x \ge -2 \}$

8. a) 16π

   b) -20

   c) 7

   d) 36π

9. a) -20

   b) $\frac{22x + 24}{33}$ or $\frac{2(11x + 12)}{33}$

10. $x^2 + (x + 1)^2 > 25$

UNIT I - IRRATIONAL NUMBERS

PURPOSE

The real number system becomes "complete" when irrational numbers are added to the integers and rationals; the property of density is fulfilled. The information of this unit can be easily adapted to right triangles and, later, to quadratic equations.

OVERVIEW

At the conclusion of this unit, the student should be able to extract roots of index 2 and greater. The student will be able to perform arithmetic and manipulative exercises with irrational numbers.

SUGGESTIONS TO THE TEACHER

Most students have not worked extensively with irrational numbers, even though a unit on square root radicals is included in Algebra 1 (see Algebra 1 Instructional Guide: Unit X).

The entering performance objectives for this unit deal only with the basic fundamentals. If results of the diagnostic test show mastery of these objectives, the teacher should move rapidly through the unit, using the third and fourth assessment from each unit objective as a guide to instruction. If performance on the diagnostic test shows need for improvement, the teacher should proceed through the unit as presented, with the knowledge that it is a review of Algebra 1 skills, extending to indices greater than two.

Computer programming in the BASIC language can be introduced at this time. (See BASIC BASIC, Coan, pp. 47-50; Algebra 2 and Trigonometry, Dolciani (1980), pp. 97-105, 175-176, 178-181, 272; Algebra Two with Trigonometry, Foster, pp. 516-536; Computer Programming in the BASIC Language, Golden, pp. 56-57: problems A 2, 4-10, B 30; Algebra Two and Trigonometry, Keedy, p. 335.)

Enrichment in approximating radicals is included at the end of the unit. The allocated time for this unit is approximately ten days.

VOCABULARY

irrational number
radicand
index
rationalize (the denominator)
root

ENTERING PERFORMANCE OBJECTIVES

1. Given a set of real numbers, select those that are irrational.
2. State a definition of the square root of a number.
3. Name in simplest form a given monomial square root radical expression.
4. Given two square root radicals, determine the product in simplest form.
5. Determine the product or quotient of a given radical expression in simplest form.
6. Given an expression involving:

   a) a product of rational expressions

   b) a quotient of rational expressions

   c) a power of a rational expression

   with positive integral exponents, use the properties of exponents to determine an equivalent expression.

DIAGNOSTIC TEST KEYED TO ENTERING PERFORMANCE OBJECTIVES

1. Select the irrational numbers from the following set:

   $\left\{17, \sqrt{2}, 4.371, \frac{2}{9}, 2\pi, 7.\overline{3}, 8\sqrt{3}, \sqrt{25}, 2.167 \ldots\right\}$

   1. ______________

2. State a definition of the square root of a number.
3. Rename each of the following in simplest form:

   a) $\sqrt{9x^2}$ a) ______________

   b) $\sqrt{\frac{4}{9}}$ b) ______________

   c) $\sqrt{18}$ c) ______________

   d) $\sqrt{\frac{3}{5}}$ d) ______________

4. Determine the product of each of the following in simplest form:

a) $\sqrt{6} \cdot \sqrt{6}$ a) ____________

b) $\sqrt{2a^3} \cdot \sqrt{8}$ b) ____________

c) $\sqrt{3} \cdot \sqrt{18}$ c) ____________

5. Determine the product or quotient in simplest form for each of the following:

a) $(\sqrt{2} + 7)(\sqrt{2} - 7)$ a) ____________

b) $(\sqrt{5} + \sqrt{7})(\sqrt{5} + \sqrt{7})$ b) ____________

c) $\dfrac{3}{\sqrt{3} + 2}$ c) ____________

d) $\dfrac{2\sqrt{3} + \sqrt{2}}{2\sqrt{3} + \sqrt{5}}$ d) ____________

6. Write in simplest form:

a) $(4x^2)(3x^5)$ a) ____________

b) $\dfrac{9x^5y}{3x^2y^2}$ b) ____________

c) $(2x^3y)^4$ c) ____________

DIAGNOSTIC TEST

ANSWERS

1. $\{\sqrt{2}, 2\pi, 8\sqrt{3}, 2.\dot{1}6\dot{7} \ldots\}$

2. The square root of a number is one of its two equal factors.

3.

a) $3x$

b) $\frac{2}{3}$

c) $3\sqrt{2}$

d) $\frac{\sqrt{15}}{5}$

4.

a) 6

b) 4

c) $3\sqrt{6}$

5.

a) -47

b) $12 + 2\sqrt{35}$ or $2(6 + \sqrt{35})$

c) $6 - 3\sqrt{3}$ or $-3(\sqrt{3} - 2)$

d) $\frac{12 - 2\sqrt{15} + 2\sqrt{6} - \sqrt{10}}{7}$

6.

a) $12x^7$

b) $\frac{3x^3}{y}$

c) $16x^{12}y^4$

UNIT I - IRRATIONAL NUMBERS

PERFORMANCE OBJECTIVES

1. State in simplest form a given monomial radical expression.

2. Given expressions involving irrational numbers, determine their sum or difference in simplest form.

3. Given an indicated product of two radicals with the same index, determine the product in simplest form.

4. Given a radical expression, determine an equivalent form by rationalizing the denominator.

5. Given expressions involving irrational numbers, determine their product in simplest form.

6. Given an indicated quotient in which the denominator is a binomial sum or difference containing a square root radical, determine the common name by rationalizing the denominator.

ENRICHMENT

Given an irrational number, determine a rational number approximation to required accuracy.

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	264-267	275-277	276-279	138-139	303-307	--	243-246 254	281-285	152-154 167-190
2	269-271	279-280	280-282	140-142	316-317	13-15	192-193 251-252 254	287-289	171-174
3	264-267	275-277	281-282	138-139	308-312	9-12	253-254	281-285	164-165
4	265-267	275-277	281-282	143-146	313-315	11-12	254-255	287-288	167-170
5	271-272	279-280	281-282	138-141	310-312 320	15	192-193 254-255	287-289	171-174
6	271-273	280-281	281-282	143-146	322-325	--	254-255	287-290	171-174
ENRICHMENT	261-263	271	273-275	136-137	310-311	--	--	280-281	158-160

PERFORMANCE OBJECTIVE I-1

State in simplest form a given monomial radical expression.

1. Rename each of the following expressions in simplest form:

   a) $\sqrt{8}$

   b) $\sqrt[3]{81}$

   c) $\sqrt{150}$

   d) $\sqrt[3]{a^3 b^4}$

   e) $\sqrt{45x^3 y^2}$

   (NOTE: 4 of 5 for mastery)

2. Rename each of the following expressions in simplest form:

   a) $\sqrt{54}$

   b) $\sqrt{72x^3}$

   c) $\sqrt{54a^4 b^3}$

   d) $\sqrt[3]{16}$

   e) $\sqrt{x^5 y^6}$

   (NOTE: 4 of 5 for mastery)

3. Rename each of the following expressions in simplest form·

   a) $\sqrt[3]{125x^3 y^2}$

   b) $\sqrt[5]{-64a^3 b^7}$

   c) $\sqrt[4]{32m^3 n^4}$

   d) $\sqrt[4]{625a^5 y^7}$

   e) $\sqrt[3]{81x^5 y^9}$

   (NOTE: 4 of 5 for mastery)

4. Rename each of the following expressions in simplest form.

a) $\sqrt[5]{-625x^3y^5z^6}$

b) $\sqrt[4]{243a^4b^6}$

c) $\sqrt[3]{216m^7n^8}$

d) $\sqrt[3]{72a^2b^9}$

e) $\sqrt[3]{686x^7y^8}$

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE I-2

Given expressions involving irrational numbers, determine their sum or difference in simplest form.

1. Determine the sum or difference, in simplest form, for each of the following expressions:

a) $8\sqrt{2} - 3\sqrt{2}$ a) ____________

b) $2\sqrt[3]{3} + 3\sqrt[3]{3}$ b) ____________

c) $2\sqrt{50} - \sqrt{72}$ c) ____________

d) $(2 + 3\sqrt{3}) - (\sqrt{3} - 6)$ d) ____________

e) $(2\sqrt{45} - \sqrt{98}) - (\sqrt{200} + \sqrt{80}$ e) ____________

(NOTE: 4 of 5 for mastery)

2. Determine the difference, in simplest form, for each of the following expressions:

a) $5\sqrt{5} - 7\sqrt{5}$ a) ____________

b) $\sqrt{8} - \sqrt{2}$ b) ____________

c) $6\sqrt{40} - 3\sqrt{10}$ c) ____________

d) $(\sqrt[3]{81} + \sqrt{8}) + (2\sqrt[3]{3} - \sqrt{32})$ d) ____________

e) $(3\sqrt{27} + \sqrt{128} - (2\sqrt{72} - \sqrt{243})$ e) ____________

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE I-2 (continued)

3. Determine the difference, in simplest form, for each of the following:

a) $5\sqrt{3} - 2\sqrt{3}$ a) ______________

b) $\sqrt{32} - \sqrt{8}$ b) ______________

c) $3\sqrt{18} - 2\sqrt{72}$ c) ______________

d) $(\sqrt[5]{32} - \sqrt[3]{16}) + (\sqrt[4]{16} + \sqrt[3]{128})$ d) ______________

e) $(2\sqrt{75} - 6\sqrt{45}) - (3\sqrt{125} - \sqrt{192})$ e) ______________

(NOTE: 4 of 5 for mastery)

4. Determine the difference in simplest form, for each of the following:

a) $7\sqrt{11} - 12\sqrt{11}$ a) ______________

b) $\sqrt{75} - 4\sqrt{3}$ b) ______________

c) $6\sqrt{27} - 2\sqrt{48}$ c) ______________

d) $(\sqrt[3]{8} + \sqrt{8}) - (\sqrt{27} + \sqrt[3]{27})$ d) ______________

e) $(2\sqrt{98} + \sqrt{243}) - (3\sqrt{128} - \sqrt{300})$ e) ______________

(NOTE: 4 out of 5 for mastery)

PERFORMANCE OBJECTIVE I-3

Given an indicated product of two radicals with the same index, determine the product in simplest form.

1. Determine the product for each of the following and express the answer in simplest form:

   a) $\sqrt{2} \cdot \sqrt{5}$ a) ______________

   b) $\sqrt{8} \cdot \sqrt{12}$ b) ______________

   c) $\sqrt[3]{64} \cdot \sqrt[3]{16}$ c) ______________

   d) $\sqrt{18x^2y} \cdot \sqrt{50xy^3}$ d) ______________

   e) $\sqrt[3]{24ab^2} \cdot \sqrt[3]{9a^2b}$ e) ______________

   (NOTE: 4 of 5 for mastery)

2. Determine the product for each of the following and express the answer in simplest form:

   a) $\sqrt{18} \cdot \sqrt{2}$ a) ______________

   b) $3\sqrt{8} \cdot 2\sqrt{98}$ b) ______________

   c) $\sqrt[3]{9} \cdot \sqrt[3]{3}$ c) ______________

   d) $\sqrt{72a^3b} \cdot 3\sqrt{45ab^3}$ dd) ______________

   e) $\sqrt[3]{36y^2z} \cdot \sqrt[3]{36x^3yz^2}$ e) ______________

   (NOTE: 4 of 5 for mastery)

3. Determine the product for each of the following and express the answer in simplest form:

a) $\sqrt[3]{-4x^2} \cdot \sqrt[3]{2x^4}$ a) ________

b) $\sqrt[5]{8x^7} \cdot \sqrt[5]{-8x^4}$ b) ________

c) $\sqrt[4]{32} \cdot \sqrt[4]{12} \cdot \sqrt[4]{54}$ c) ________

d) $\sqrt{128m^3n^5} \cdot \sqrt{63m^2n^3}$ d) ________

e) $2\sqrt[3]{49x^2} \cdot 3\sqrt[3]{21x^2y^3}$ e) ________

(NOTE: 4 of 5 for mastery)

4. Determine the product for each of the following and express the answer in simplest form:

a) $\sqrt{45} \cdot \sqrt{80}$ a) ________

b) $2\sqrt{8} \cdot 3\sqrt{32}$ b) ________

c) $\sqrt[3]{14} \cdot \sqrt[3]{49}$ c) ________

d) $\sqrt[4]{32a^3b^2} \cdot \sqrt[4]{81a^2b}$ d) ________

e) $3\sqrt{18x^4y} \cdot 2\sqrt[3]{12xy^2}$ e) ________

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE I-4

Given a radical expression, determine an equivalent form by rationalizing the denominator.

1. Rationalize the denominator for each of the following fractions:

a) $\frac{1}{\sqrt{2}}$ a) ____________

b) $\sqrt{\frac{2}{9}}$ b) ____________

c) $\frac{18}{3\sqrt{12}}$ c) ____________

d) $\frac{1}{\sqrt[3]{5}}$ d) ____________

e) $\sqrt[3]{\frac{2}{9}}$ e) ____________

(NOTE: 4 of 5 for mastery)

2. Rationalize the denominator for each of the following fractions:

a) $\frac{1}{\sqrt{5}}$ a) ____________

b) $\sqrt{\frac{3}{16}}$ b) ____________

c) $\frac{5}{2\sqrt{\text{[illegible]}}}$ c) ____________

d) $\frac{1}{\sqrt[3]{2}}$ d) ____________

e) $\sqrt[3]{\frac{3}{4}}$ e) ____________

(NOTE: 4 of 5 for mastery)

3. Rationalize the denominator for each of the following fractions:

a) $\sqrt{\frac{7}{4}}$ a) ____________

b) $\frac{13}{\sqrt{13}}$ b) ____________

c) $\frac{2x}{\sqrt{3x^3}}$ c) ____________

d) $\sqrt[3]{\frac{2}{5}}$ d) ____________

e) $\frac{6}{\sqrt[4]{3}}$ e) ____________

(NOTE: 4 of 5 for mastery)

4. Rationalize the denominator for each of the following fractions:

a) $\frac{3}{\sqrt{2}}$ a) ____________

b) $\frac{\sqrt{3}}{\sqrt{4}}$ b) ____________

c) $\frac{2\sqrt{5}}{5\sqrt{2}}$ c) ____________

d) $\frac{2\sqrt[4]{5}}{5\sqrt[4]{2x^2}}$ d) ____________

e) $\sqrt[3]{\frac{7}{16}}$ e) ____________

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE I-5

Given expressions involving irrational numbers, determine their product in simplest form.

1. Determine the product of each of the following and express the answer in simplest form:

a) $(2 + \sqrt{3})(2 - \sqrt{3})$ a) ________

b) $(\sqrt{6} + \sqrt{5})(\sqrt{6} + \sqrt{5})$ b) ________

c) $(2\sqrt{6} - 3)(7 - \sqrt{6})$ c) ________

d) $(\sqrt{12} - \sqrt{18})(\sqrt{27} + \sqrt{8})$ d) ________

e) $\left(\frac{-4 + \sqrt{2}}{3}\right)\left(\frac{-4 - \sqrt{2}}{3}\right)$ e) ________

(NOTE: 4 of 5 for mastery)

2. Determine the product of each of the following and express the answer in simplest form:

a) $\sqrt{3}(2\sqrt{3} - 5\sqrt{2})$ a) ________

b) $(1 - \sqrt{2})(1 + \sqrt{2})$ b) ________

c) $(\sqrt{7} + \sqrt{3})(\sqrt{7} + \sqrt{3})$ c) ________

d) $(\sqrt{98} - \sqrt{12})(\sqrt{75} - \sqrt{18})$ d) ________

e) $\left(\frac{-3 + \sqrt{2}}{6}\right)\left(\frac{-3 - \sqrt{2}}{6}\right)$ e) ________

(NOTE: 4 of 5 for mastery)

3. Determine the product of each of the following and express the answer in simplest form:

a) $\sqrt[3]{2}(6\sqrt[3]{3} - 3\sqrt[3]{2})$ a) ____________

b) $(4x\sqrt{3} - 2\sqrt{5})(4x\sqrt{3} + 2\sqrt{5})$ b) ____________

c) $(\sqrt{7} + \sqrt{6})(\sqrt{7} + \sqrt{6})$ c) ____________

d) $(\sqrt{8} + \sqrt{48})\ (\sqrt{108} + \sqrt{32})$ d) ____________

e) $\left(\frac{-3 + \sqrt{2}}{6}\right)\left(\frac{-3 - \sqrt{2}}{6}\right)$ e) ____________

(NOTE: 4 of 5 for mastery)

4. Determine the product of each of the following and express the answer in simplest form:

a) $\sqrt{5}\ (3\sqrt{2} - 5\sqrt{5})$ a) ____________

b) $(6 + \sqrt{2})\ (6 - \sqrt{2})$ b) ____________

c) $(2\sqrt{3} + \sqrt{7})\ (2\sqrt{3} + \sqrt{7})$ c) ____________

d) $(\sqrt{128} - \sqrt{24})\ (\sqrt{54} - \sqrt{32})$ d) ____________

e) $\left(\frac{-5 - 4\sqrt{3}}{7}\right)\left(\frac{-5 + 4\sqrt{3}}{7}\right)$ e) ____________

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE I-6

Given an indicated quotient in which the denominator is a binomial sum or difference containing a square root radical, determine the common name by rationalizing the denominator.

1. Simplify each of the following by rationalizing the denominator:

a) $\frac{1}{3-\sqrt{2}}$ a) ____________

b) $\frac{7}{\sqrt{3}+5}$ b) ____________

c) $\frac{\sqrt{7}}{\sqrt{5}-\sqrt{3}}$ c) ____________

d) $\frac{\sqrt{6}-2}{1+\sqrt{6}}$ d) ____________

e) $\frac{\sqrt{5}-\sqrt{x}}{\sqrt{5}+\sqrt{x}}$ e) ____________

(NOTE: 4 of 5 for mastery)

2. Simplify each of the following by rationalizing the denominator:

a) $\frac{1}{\sqrt{2}+3}$ a) ____________

b) $\frac{8}{\sqrt{3}-1}$ b) ____________

c) $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{5}}$ c) ____________

d) $\frac{\sqrt{7}-4}{2+\sqrt{7}}$ d) ____________

e) $\frac{\sqrt{2}+\sqrt{x}}{\sqrt{2}-\sqrt{x}}$ e) ____________

(NOTE: 4 of 5 for mastery)

3. Simplify each of the following by rationalizing the denominator:

a) $\frac{1}{\sqrt{7} - 4}$ a) ____________

b) $\frac{5}{2 - \sqrt{6}}$ b) ____________

c) $\frac{\sqrt{10}}{\sqrt{6} - \sqrt{2}}$ c) ____________

d) $\frac{\sqrt{3} - 1}{6 + \sqrt{3}}$ d) ____________

e) $\frac{\sqrt{2} + \sqrt{y}}{\sqrt{y} - \sqrt{2}}$ e) ____________

(NOTE: 4 of 5 for mastery)

4. Simplify each of the following by rationalizing the denominator:

a) $\frac{1}{\sqrt{5} + 3}$ a) ____________

b) $\frac{12}{7 - \sqrt{6}}$ b) ____________

c) $\frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}}$ c) ____________

d) $\frac{\sqrt{5} + 4}{3 - \sqrt{5}}$ d) ____________

e) $\frac{\sqrt{3} - \sqrt{a}}{\sqrt{3} + \sqrt{a}}$ e) ____________

ENRICHMENT

1. Determine $\sqrt{7}$ to the nearest hundredth.
2. Determine $\sqrt{27}$ to the nearest hundredth.
3. Determine $\sqrt{603.4}$ to the nearest tenth.
4. Determine $\sqrt{2035}$ to the nearest tenth.
5. Determine $\sqrt[3]{72}$ to the nearest tenth.

ANSWERS

I-1

1. a) $2\sqrt{2}$
   b) $3\sqrt[3]{3}$
   c) $5\sqrt{6}$
   d) $ab\sqrt[3]{b}$
   e) $3x\,|y|\sqrt{5x}$ or $3xy\sqrt{5x}$

2. a) $3\sqrt{6}$
   b) $6x\sqrt{2x}$
   c) $3ab\sqrt[3]{2a}$
   d. $2\sqrt[3]{2}$
   e. $x^2\,|y^3|\sqrt{x}$ or $x^2y^3\sqrt{x}$

3. a) $5x\sqrt[3]{y^2}$
   b) $-2b\sqrt[5]{2a^3b^2}$
   c) $2\,|n|\sqrt[4]{2m^3}$ or $2n\sqrt[4]{2m^3}$
   d) $5ab\sqrt[4]{ab^3}$
   e) $3xy^3\sqrt[3]{3x^2}$

4. a) $-5xyz^2\sqrt[3]{5y^2}$
   b) $3\,|ab|\sqrt[4]{3b^2}$ or $3ab\sqrt[4]{3b^2}$
   c) $6m^2n^2\sqrt[3]{m}$
   d) $2b^3\sqrt[3]{9a^2}$
   e) $7x^2y^3\sqrt[3]{2xy^3}$

I-2

1. a) $5\sqrt{2}$
   b) $5\sqrt[3]{3}$
   c) $4\sqrt{2}$
   d) $8 + 2\sqrt{3}$ or $2(4 + \sqrt{3})$
   e) $2\sqrt{5} - 17\sqrt{2}$

2. a) $-2\sqrt{5}$
   b) $\sqrt{2}$
   c) $9\sqrt{10}$
   d) $5\sqrt[3]{3} - 2\sqrt{2}$
   e) $-4\sqrt{2}$

3. a) $3\sqrt{3}$
   b) $2\sqrt{2}$
   c) $-3\sqrt{2}$
   d) $2(2 + \sqrt[3]{2})$
   e) $18\sqrt{3} - 33\sqrt{5}$

4. a) $-5\sqrt{11}$
   b) $\sqrt{3}$
   c) $10\sqrt{3}$
   d) $-1 + 2\sqrt{2} + 3\sqrt{3}$
   e) $-10\sqrt{2} + 19\sqrt{3}$

ANSWERS

I-3

1. a) $\sqrt{10}$
   b) $4\sqrt{6}$
   c) $8\sqrt[3]{2}$
   d) $30 x y^2\sqrt{x}$
   e) $6ab$

2. a) 6
   b) 168
   c) 3
   d) $54a^2b^2\sqrt{10}$
   e) $6xyz\sqrt[3]{6}$

3. a) $-2x^2$
   b) $-2x^2\sqrt[5]{2x}$
   c) 12
   d) $24m^2n^4\sqrt{14m}$
   e) $42xy\sqrt[3]{3x}$

4. a) 60
   b) 96
   c) $7\sqrt[3]{2}$
   d) $6a\sqrt[4]{2ab^3}$
   e) $36xy\sqrt[3]{x^2}$

I-4

1. a) $\frac{\sqrt{2}}{2}$
   b) $\frac{\sqrt{2}}{3}$
   c) $\sqrt{3}$
   d) $\frac{\sqrt[3]{25}}{5}$
   e) $\frac{\sqrt[3]{6}}{3}$

2. a) $\frac{\sqrt{5}}{5}$
   b) $\frac{\sqrt{3}}{4}$
   c) $\frac{\sqrt{2}}{4}$
   d) $\frac{\sqrt[3]{4}}{2}$
   e) $\frac{\sqrt[3]{6}}{2}$

3. a) $\sqrt{\frac{7}{2}}$
   b) $\sqrt{13}$
   c) $\frac{2\sqrt{3x}}{3x}$
   d) $\frac{-\sqrt[3]{50}}{5}$
   e) $3\sqrt{2}$

ANSWERS

I-4 (continued)

4. a) $\frac{3\sqrt{2}}{2}$

b) $\frac{\sqrt{3}}{2}$

c) $\frac{\sqrt[3]{15}}{5}$

d) $\frac{\sqrt[4]{40x^2}}{5x}$

e) $\frac{\sqrt[3]{28}}{4}$

I-5

1. a) 1

b) $11 + 2\sqrt{30}$

c) $-33 + 17\sqrt{6}$

d) $6 - 5\sqrt{6}$

e) $\frac{14}{9}$

2. a) $6 - 5\sqrt{6}$

b) -1

c) $10 + 2\sqrt{21}$

d) $-72 + 41\sqrt{6}$

e) $\frac{31}{9}$

I-5 (continued)

3. a) $6\sqrt[3]{6} - 3\sqrt[3]{4}$

b) $48x^2 - 20$

c) $13 + 2\sqrt{42}$

d) $88 + 28\sqrt{6}$

e) $\frac{9 - \sqrt[5]{4}}{36x^2}$

4. a) $-25 + 3\sqrt{10}$

b) $36 - \sqrt[3]{4}$

c) $12 + 4\sqrt{21x} + 7x$

d) $-100 + 64\sqrt{3}$

e) $\frac{-23}{49}$

I-6

1. a) $\frac{3 + \sqrt{2}}{7}$

b) $\frac{35 - 7\sqrt{3}}{22}$ or $\frac{7}{22}(5 - \sqrt{3})$

c) $\frac{\sqrt{35} + \sqrt{21}}{2}$

d) $\frac{8 - 3\sqrt{6}}{5}$

e) $\frac{5 - 2\sqrt{5x} + x}{5-x}$

ANSWERS

I-6 (continued)

2. a) $\frac{3 - \sqrt{2}}{7}$

b) $4 + 4\sqrt{3}$

c) $2\sqrt{3} + \sqrt{10}$

d) $5 - 2\sqrt{7}$

e) $\frac{2 + 2\sqrt{2x} + x}{2-x}$

3. a) $\frac{-4 - \sqrt{7}}{9}$

b) $\frac{-10 - 5\sqrt{6}}{2}$

c) $\frac{\sqrt{15} + \sqrt{5}}{2}$

d) $\frac{-9 + 7\sqrt{3}}{33}$

e) $\frac{y + 2\sqrt{2y} + 2}{y - 2}$

4. a) $\frac{3 - \sqrt{5}}{4}$

b) $\frac{84 + 12\sqrt{6}}{43}$

c) $-2 + \sqrt{6}$

d) $\frac{17 + 7\sqrt{5}}{4}$

e) $\frac{3 - 2\sqrt{3a} - a}{3 - a}$

ENRICHMENT

1. 2.65
2. 5.20
3. 24.6
4. 45.1
5. 4.2

UNIT II - COMPLEX NUMBERS

PURPOSE

This unit is designed to advance students' knowledge of number systems. A new number system is developed with which the square root of a negative real number may be determined. If this unit is taught early in the course, the class may then use complex numbers and their properties in remaining units.

OVERVIEW

Whole numbers contain the additive identity, a property lacking in the natural numbers. Integers possess additive inverses; rational numbers contain multiplicative inverses; real numbers contain square roots. Thus, a natural introductory question for this unit would be, "What is the $\sqrt{-4}$?" Accordingly, the concept of the pure imaginary unit is developed. Subsequently, the complex number system is fully developed.

Upon completion of this unit, the student should be able to: (1) identify a complex number and its components; (2) perform arithmetic operations using complex numbers; and, (3) solve a linear equation containing complex numbers.

SUGGESTIONS TO THE TEACHER

In order to be successful in this unit, the student should have mastered the skills in Unit I, "Irrational Numbers." There are no other entering performance objectives for this unit.

It is important to emphasize the difference between a complex number and a pure imaginary number throughout the unit.

There are fifteen performance objectives in the unit, and they should be completed in approximately ten days.

Computer Applications: BASIC BASIC, Coan, pp. 139-141; Computer Programming in the BASIC Language, Golden, p. 85; Algebra Two with Trigonometry, Payne, pp. 518-519; Algebra Two and Trigonometry, Keedy, p. 361.

VOCABULARY

pure imaginary number
complex number
real number part
imaginary number part
complex conjugate

UNIT II - COMPLEX NUMBERS

PERFORMANCE OBJECTIVES

1. Given a set of numbers, identify the pure imaginary numbers.
2. Given the square root of a negative number, write it as a pure imaginary number in simplest i-form.
3. Determine i^n for any positive integer n.
4. Given an expression involving either addition or subtraction of pure imaginary numbers, determine the solution in i-form.
5. Demonstrate the procedure for finding the product of two pure imaginary numbers.
6. Given an expression involving division of pure imaginary numbers, determine the quotient in simplest form.
7. Given a complex number, identify the real and imaginary parts.
8. Given an expression involving either addition or subtraction of complex numbers, determine the solution in a + bi form.
9. Given a complex number, determine its additive inverse.
10. Determine the coefficients of the real number and imaginary number parts, given basic linear equations.
11. Given an expression involving multiplication of complex numbers, determine the product in a + bi form.
12. Given a complex number, name its conjugate.
13. Determine the product of two conjugate complex numbers.
14. Determine the reciprocal of a given complex number.
15. Demonstrate the procedure for finding the quotient of two complex numbers.

ENRICHMENT

1. Given a complex number, determine its absolute value.
2. Apply the properties of a group to imaginary numbers.
3. Construct a geometric model associated with a given complex number.

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	410-411	292	295-299	150-152	342-343	--	229	291-293	176-178
2	411-413	292-295	295-299	150-152	342-343	15-16 18	225-226	291-293	176-179
3	411-414	292-293	297	151-152	345	19	225-227	292-293	176-179
4	411-414	293-294	297-299	151-152	343-344	17-18	227-228	291-293	176-179
5	411-413	293-294	297-299	151-152	346-349	17-19	227-228	292-293	176-179
6	412-414	293-294	297-299	--	--	18-19	227	292-293	--
7	415-416	295	299-302	154-156	343	20	229	294-295	178
8	415-417	296-297	300-302	154-156	344-345	23-25	232-235	294-297	183-185
9	--	296-297	300-302	159-161	344-345	23-24	234	298	183
10	408-410	297	--	154-156	346	--	--	294-296	--

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
11	414-417	298-299	302-304	154-156	346-347	26-28	233-235	294-296	187-190
12	414,417	296	300-302	155-156	347-349	29-31	236-239	297-298	--
13	415	298	302-304	155-156	346-350	29	236-239	297-298	--
14	--	299	302-304	160	348	29-30	237	297-299	--
15	414-417	299-300	302-304	157-158	348-350	29-32	236-239	297-299	188-190
ENRICHMENT 1	--	566	598	--	355-356	168	241-243	466-467	181
2	--	--	--	--	12 344	43 48	42-45	--	540(cf
3	--	566	598	--	355-356	163-169	474-476	--	--

PERFORMANCE OBJECTIVE II-1

Given a set of numbers, identify the pure imaginary numbers.

1. Identify the pure imaginary numbers in the following set:

$\left\{7,\ 6i,\ \pi,\ \sqrt{3},\ 14,\ \sqrt{-9},\ \frac{\pi}{3},\ \sqrt{-1}\right\}$.

1. ____________

2. Identify the pure imaginary numbers in the following set:

$\left\{\sqrt{5},\ 3\pi,\ 3\sqrt{-2},\ 18,\ 12i,\ \frac{2\pi}{3},\ \sqrt{-16},\ 2\right\}$.

2. ____________

3. Identify the pure imaginary numbers in the following set:

$\left\{11,\ \sqrt{-2},\ 3\sqrt{7},\ 2\pi,\ 9i,\ \sqrt{-10},\ \frac{\pi}{4},\ 3\right\}$.

3. ____________

4. Identify the pure imaginary numbers in the following set:

$\left\{3\sqrt{-3},\ \sqrt{6},\ 4\pi,\ 7i,\ 19,\ \sqrt{-1},\ 4,\ \frac{\pi}{3}\right\}$.

4. ____________

PERFORMANCE OBJECTIVE II-2

Given the square root of a negative number, write it as a pure imaginary number in simplest i-form.

1. Rename each of the following in i-form:

a) $\sqrt{-1}$ a) ______

b) $\sqrt{-9}$ b) ______

c) $\sqrt{-8}$ c) ______

d) $\sqrt{-108}$ d) ______

e) $\sqrt{-\frac{1}{3}}$ e) ______

(NOTE: 4 of 5 for mastery)

2. Rename each of the following in i-form:

a) $\sqrt{-16}$ a) ______

b) $\sqrt{-45}$ b) ______

c) $\sqrt{-2}$ c) ______

d) $\sqrt{-243}$ d) ______

e) $\sqrt{-\frac{1}{2}}$ e) ______

(NOTE: 4 of 5 for mastery)

3. Rename each of the following in i-form:

a) $\sqrt{-4}$ a) ______

b) $\sqrt{-5}$ b) ______

c) $\sqrt{-50}$ c) ______

d) $\sqrt{-432}$ d) ______

e) $\sqrt{-\frac{1}{5}}$ e) ______

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE II-2 (continued)

4. Rename each of the following in i-form:

a) $\sqrt{-25}$ a) ______________

b) $\sqrt{-7}$ b) ______________

c) $\sqrt{-32}$ c) ______________

d) $\sqrt{-192}$ d) ______________

e) $\sqrt{-\frac{1}{8}}$ e) ______________

PERFORMANCE OBJECTIVE II-3

Determine i^n for any positive integer n.

1. i^{17} is equivalent to

   a) i

   b) $-i$

   c) 1

   d) -1

2. i^{32} is equivalent to

   a) i

   b) $-i$

   c) 1

   d) -1

3. i^{67} is equivalent to

   a) i

   b) $-i$

   c) 1

   d) -1

4. i^{42} is equivalent to

   a) i

   b) $-i$

   c) 1

   d) -1

PERFORMANCE OBJECTIVE II-4

Given an expression involving either addition or subtraction of pure imaginary numbers, determine the solution in i-form.

1. $\sqrt{-25} - \sqrt{-4}$ is equivalent to

   a) 3

   b) -3

   c) 3i

   d) $\sqrt{-21}$

   e) $-\sqrt{-29}$

2. $5\sqrt{-3} - \sqrt{-27}$ is equivalent to

   a) $5i\sqrt{3} + 3\sqrt{3}$

   b) $8i\sqrt{3}$

   c) 2i

   d) $2i\sqrt{3}$

   e) 8i

3. $\sqrt{-20} + \sqrt{-8}$ is equivalent to

   a) $4i\sqrt{7}$

   b) $2(\sqrt{5} + \sqrt{2})i$

   c) $4i(\sqrt{5} + \sqrt{2})$

   d) $2i\sqrt{7}$

   e) $4\sqrt{7}$

4. $\sqrt{-\frac{3}{4}} - \sqrt{-\frac{1}{2}}$ is equivalent to

   a) $\frac{i\sqrt{3}}{4} - \frac{i}{2}$

   b) $-\frac{i}{2}$

   c) $\left(\frac{\sqrt{3} - \sqrt{2}}{2}\right)i$

   d) $\left(\frac{\sqrt{3} - \sqrt{2}}{4}\right)i$

   e) $\frac{i\sqrt{3}}{4} - \frac{i\sqrt{2}}{2}$

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PERFORMANCE OBJECTIVE II-5

Demonstrate the procedure for finding the product of two pure imaginary numbers.

1. Demonstrate the procedure for finding the product of $\sqrt{-2}$ and $\sqrt{-3}$.

2. Demonstrate the procedure for finding the product of $\sqrt{-8}$ and $\sqrt{-16}$.

3. Demonstrate the procedure for finding the product of $\sqrt{-32}$ and $\sqrt{-45}$.

4. Demonstrate the procedure for finding the product of $\sqrt{-75}$ and $\sqrt{-432}$.

PERFORMANCE OBJECTIVE II-6

Given an expression involving division of pure imaginary numbers, determine the quotient in simplest form.

1. $\frac{\sqrt{-12}}{\sqrt{-2}}$ is equivalent to

   a) 6

   b) $i\sqrt{6}$

   c) $\sqrt{6}$

   d) $6i$

2. $\frac{\sqrt{-2}}{\sqrt{-8}}$ is equivalent to

   a) $\frac{i}{2}$

   b) $\frac{1}{2}$

   c) $-\frac{1}{2}$

   d) $\sqrt{\frac{1}{2}}$

3. $\frac{6\sqrt{-3}}{5\sqrt{-2}}$ is equivalent to

   a) $\frac{3\sqrt{6}}{5}$

   b) $\frac{3\sqrt{3}}{5}$

   c) $\frac{6i\sqrt{3}}{5}$

   d) $\frac{3i\sqrt{6}}{5}$

4. $\frac{\sqrt{-20}}{2\sqrt{-5}}$ is equivalent to

   a) $\frac{2i\sqrt{2}}{5}$

   b) i

   c) 1

   d) $2i$

PERFORMANCE OBJECTIVE II-7

Given a complex number, identify the real and imaginary parts.

1. 2 + 3i

   a) The real part is: ______________

   b) The imaginary part is: ______________

2. 6 - 4i

   a) The real part is: ______________

   b) The imaginary part is ______________

3. 7

   a) The real part is: ______________

   b) The imaginary part is ______________

4. -9i

   a) The real part is: ______________

   b) The imaginary part is: ______________

PERFORMANCE OBJECTIVE II-8

Given an expression involving either addition or subtraction of complex numbers, determine the solution in a + bi form.

1. $(6 - 2i) + (3 + 7i)$ is equivalent to

   a) $3 + 5i$

   b) $9 + 5i$

   c) $9 - 5i$

   d) $3 - 5i$

   e) $9 - 9i$

2. $(5 - 3i) - (8 + 2i)$ is equivalent to

   a) $-3 - 5i$

   b) $13 - 5i$

   c) $-3 + 5i$

   d) $-3 - i$

   e) $3 - i$

3. $(4 + 2i) - (3 - 4i)$ is equivalent to

   a) $7 + 2i$

   b) $1 - 2i$

   c) $1 + 6i$

   d) $7 + 6i$

   e) $-1 + 6i$

4. $(a + bi) + (c + di)$ is equivalent to

   a) $(a + b) + (c + d)i$

   b) $(b + c) + (a + d)i$

   c) $(a + d) + (b + c)i$

   d) $(a + c) + (b + d)i$

   e) $(b + d) + (a + c)i$

PERFORMANCE OBJECTIVE II-9

Given a complex number, determine its additive inverse.

1. Determine the additive inverse of:

   a) $6 + 2i$

   b) $7 - 3i$

   c) $-4 + i$

   d) $-5i$

   e) $\sqrt{6} + i\sqrt{7}$

   (NOTE: 4 of 5 for mastery)

2. Determine the additive inverse of:

   a) $2 + 7i$

   b) $4 - i$

   c) $6i$

   d) $-9 - 2i$

   e) $\sqrt{3} - i\sqrt{2}$

   (NOTE: 4 of 5 for mastery)

3. Determine the additive inverse of:

   a) $4 + 5i$

   b) $3 - 7i$

   c) $-1 + 9i$

   d) $8i$

   e) $\sqrt{5} - i\sqrt{11}$

   (NOTE: 4 of 5 for mastery)

4. Determine the additive inverse of:

   a) $3 + 2i$

   b) 4

   c) -9

   d) $-8 - 5i$

   e) $\sqrt{2} - i\sqrt{3}$

   (NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE II-10

Determine the coefficients of the real number and imaginary number parts, given basic linear equations.

1. Determine the values of x and y for each of the following:

   a) $x + yi = 3 + 2i$

   b) $x + yi - 5 + 4i = 0$

   c) $2x + yi - 7 = x + 5i$

   d) $(5 + 3i) - (2 + i) + (9 - 3i) = x + yi$

   e) $4x + (y - 2)i + 8i = 12$

   (NOTE: 4 of 5 for mastery)

2. Determine the values of x and y for each of the following:

   a) $x + yi = 7 - 3i$

   b) $x - yi + 6i + 5 = 0$

   c) $3x + 5 + yi = x + 1 - 7i$

   d) $(9 - 7i) + (x + yi) - 3i = 10$

   e) $2x - (y + 3)i = -14 + 3i$

   (NOTE: 4 of 5 for mastery)

3. Determine the values of x and y for each of the following:

   a) $x + yi = -5 + 2i$

   b) $x - 5i + 7 + yi = 4$

   c) $4x + 3i - 1 = yi + 11 - 2x$

   d) $(4 - i) + (5 - 3i) + (2x + yi) = -7 + 2i$

   e) $(x + 5) - (y + 1)i = -5i$

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PERFORMANCE OBJECTIVE II-10 (continued)

4. Determine the values of x and y for each of the following:

a) $x + yi = -3 - i$

b) $x + 3i = -10 - yi$

c) $i - 3x = 6yi + 5x - 16$

d) $(-5 + 3i) + (6 - i) - (-2 + 5i) = x + 3yi$

e) $7x + (y - 1)i + 2x = -x + 10i$

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE II-11

Given an expression involving multiplication of complex numbers, determine the product in a + bi form.

1. Determine the product of $(4 + 6i)(3 - 2i)$.
2. Determine the product of $(5 - 6i)(1 + 0i)$.
3. Determine the product of $(3 + 5i)(7 + i)$.
4. Determine the product of $(1 - 3i)(2 - 2i)$.

PERFORMANCE OBJECTIVE II-12

Given a complex number, name its conjugate.

1. Name the conjugate of $6 + 5i$.
2. Name the conjugate of $7 - 3i$.
3. Name the conjugate of 4.
4. Name the conjugate of $-2i$.

PERFORMANCE OBJECTIVE II-13

Determine the product of two conjugate complex numbers.

1. Determine the product of $(6 - 3i)(6 + 3i)$ and express the answer in simplest form.
2. Determine the product of $(-4 + i)(-4 - i)$ and express the answer in simplest form.
3. Determine the product of $(0 - 2i)(0 + 2i)$ and express the answer in simplest form.
4. Determine the product of $(a + bi)(a - bi)$ and express the answer in simplest form.

PERFORMANCE OBJECTIVE II-14

Determine the reciprocal of a given complex number.

1. The reciprocal of $1-i$ is
   a) $1 + i$
   b) $i + \frac{i}{2}$
   c) $\frac{1}{2} - \frac{1}{2}i$
   d) $\frac{1}{2} + \frac{1}{2}i$

2. The reciprocal of $3 + 4i$ is
   a) $\frac{3}{25} - \frac{4}{25}i$
   b) $3 - 4i$
   c) $-\frac{3}{7} + \frac{4i}{7}$
   d) $\frac{3}{7} - \frac{4i}{7}$

3. The reciprocal of $-2i$ is
   a) $2i$
   b) $\frac{i}{2}$
   c) $\frac{1}{2}$
   d) $\frac{i}{4}$

4. The reciprocal of $a + bi$ is
   a) $\frac{a}{a^2 + b^2} + \frac{bi}{a^2 + b^2}$
   b) $\frac{a}{a^2 - b^2} + \frac{bi}{a^2 - b^2}$
   c) $\frac{a}{a^2 + b^2} - \frac{bi}{a^2 + b^2}$
   d) $\frac{a}{a^2 - b^2} - \frac{bi}{a^2 - b^2}$

PERFORMANCE OBJECTIVE II-15

Demonstrate the procedure for finding the quotient of two complex numbers.

1. Demonstrate the procedure for finding the quotient $\frac{5}{\sqrt{-8}}$. Express the answer in standard form.

2. Demonstrate the procedure for finding the quotient $\frac{3i}{2 + 3i}$. Express the answer in a + bi form.

3. Demonstrate the procedure for finding the quotient $\frac{1 - 2i}{4 + 3i}$. Express the answer in a + bi form.

4. Demonstrate the procedure for finding the quotient $\frac{3 + \sqrt{-2}}{4 - \sqrt{-2}}$. Express the answer in a + bi form.

ENRICHMENT 1

1. $|3 - 4i|$ is equivalent to:

   a) $3 + 4i$

   b) $i\sqrt{7}$

   c) 5

   d) 25

   e) -7

2. $|8 + 6i|$ is equivalent to:

   a) 100

   b) $2\sqrt{7}$

   c) $10i$

   d) 10

   e) $8 + 6i$

3. $|9 - 12i|$ is equivalent to:

   a) 15

   b) $3i\sqrt{7}$

   c) $15i$

   d) 225

   e) $9 + 12i$

4. $|3x + 2iy|$ is equivalent to:

   a) $3x^2 + 2y^2$

   b) $3x + 2y$

   c) $\sqrt{9x^2 + 4y^2}$

   d) $\sqrt{9x^2 - 4y^2}$

   e) $3x + 2iy$

ENRICHMENT 2

1. Prove that the set 1, -1, i -i forms a group under multiplication.

2. Show that A. is a commutative group.

ENRICHMENT 3

Graph the following complex numbers:

1. $3 + 5i$

2. $-2 + i$

3. $3i - 2$

ANSWERS

II-1

1. $\{6i, \sqrt{-9}, \sqrt{-1}\}$

2. $\{3\sqrt{-2}, 12i, \sqrt{-16}\}$

3. $\{\sqrt{-2}, 9i, \sqrt{-10}\}$

4. $\{3\sqrt{-3}, 7i, \sqrt{-1}\}$

II-2

1. (a) i
   (b) $3i$
   (c) $2i\sqrt{2}$
   (d) $6i\sqrt{3}$
   (e) $\frac{i\sqrt{3}}{3}$

2. (a) $4i$
   (b) $3i\sqrt{5}$
   (c) $i\sqrt{2}$
   (d) $9i\sqrt{3}$
   (e) $\frac{i\sqrt{2}}{2}$

3. (a) $2i$
   (b) $i\sqrt{5}$
   (c) $5i\sqrt{2}$
   (d) $12i\sqrt{3}$
   (e) $\frac{i\sqrt{5}}{5}$

II-2 (continued)

4. (a) $5i$
   (b) $i\sqrt{7}$
   (c) $4i\sqrt{2}$
   (d) $8i\sqrt{3}$
   (e) $\frac{i\sqrt{2}}{4}$

II-3

1. a
2. c
3. b
4. d

II-4

1. c
2. d
3. b
4. c

II-5

1. $\sqrt{-2} \cdot \sqrt{-3} = i\sqrt{2} \cdot i\sqrt{3}$
$= i^2 \sqrt{6}$
$(= (-1) \cdot \sqrt{6})$
$= -\sqrt{6}$

ANSWERS

II-5 (continued)

2. $\sqrt{-8} \cdot \sqrt{-16} = 2i\sqrt{2} \cdot 4i$

$= 8i^2\sqrt{2}$

$(= 8(-1)\sqrt{2})$

$= -8\sqrt{2}$

3. $\sqrt{-32} \cdot \sqrt{-45} = 4i\sqrt{2} \cdot 3i\sqrt{5}$

$= 12i^2\sqrt{10}$

$(= 12(-1)\sqrt{10})$

$= -12\sqrt{10}$

4. $\sqrt{-75} \cdot \sqrt{-432} = 5i\sqrt{3} \cdot 12i\sqrt{3}$

$= 60i^2 \cdot 3$

$(= 180(-1))$

$= -180$

II-6

1. c
2. b
3. a
4. c

II-7

1. a) 2
   b) $3i$
2. a) 6
   b) $-4i$

II-7 (continued)

3. a) 7
   b) $0i$
4. a) 0
   b) $-9i$

II-8

1. b
2. a
3. c
4. d

II-9

1. a) $-6 - 2i$
   b) $-7 + 3i$
   c) $4 - i$
   d) $5i$
   e) $-\sqrt{6} - i\sqrt{7}$
2. a) $-2 - 7i$
   b) $-4 + i$
   c) $-6i$
   d) $9 + 2i$
   e) $-\sqrt{3} + i\sqrt{2}$

ANSWERS

II-9 (continued)

3. a) $-4 - 5i$
   b) $-3 + 7i$
   c) $1 - 9i$
   d) $-8i$
   e) $-\sqrt{5} + i\sqrt{11}$

4. a) $-3 - 2i$
   b) $-4 + 7i$
   c) 9
   d) $8 + 5i$
   e) $-\sqrt{2} + i\sqrt{3}$

II-10 (continued)

3. a) $x = -5,\ y = 2$
   b) $x = -3,\ y = 5$
   c) $x = 2,\ y = 3$
   d) $x = -8,\ y = 6$
   e) $x = -5,\ y = 4$

4. a) $x = -3,\ y = -1$
   b) $x = -10,\ y = -3$
   c) $x = 2,\ y = \frac{1}{6}$
   d) $x = 3,\ y = -1$
   e) $x = 0,\ y = 11$

II-10

1. a) $x = 3,\ y = 2$
   b) $x = 5,\ y = -4$
   c) $x = 7,\ y = 5$
   d) $x = 12,\ y = -1$
   e) $x = 3,\ y = -6$

2. a) $x = 7,\ y = -3$
   b) $x = -5,\ y = 6$
   c) $x = -2,\ y = -7$
   d) $x = 1,\ y = 10$
   e) $x = -7,\ y = -6$

II-11

1. $24 + 10i$
2. $5 - 6i$
3. $-35 + 21i$
4. $-4 - 8i$

II-12

1. $6 - 5i$
2. $7 + 3i$
3. 4
4. $2i$

ANSWERS

II-13

1. 45

2. 17

3. 4

4. $a^2 + b^2$

II-14

1. d

2. a

3. e

4. c

II-15

1. $\frac{5}{\sqrt{-8}} = \frac{5}{2i\sqrt{2}}$

$= \frac{5}{2i\sqrt{2}} \cdot \frac{i\sqrt{2}}{i\sqrt{2}}$

$= \frac{5i\sqrt{2}}{4i^2}$

$\left(= \frac{5i\sqrt{2}}{4(-1)} \right)$

$= \frac{5i\sqrt{2}}{-4}$

II-15 (continued)

2. $\frac{3i}{2 + 3i} = \frac{3i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i}$

$= \frac{6i - 9i^2}{4 + 9}$

$\left(= \frac{-9(-1) + 6i}{13} \right)$

$= \frac{9 + 6i}{13}$

$= \frac{9}{13} + \frac{6i}{13}$

3. $\frac{1 - 2i}{4 + 3i} = \frac{1 - 2i}{4 + 3i} \cdot \frac{4 - 3i}{4 - 3i}$

$= \frac{(4 - 6) + (-3 - 8)i}{16 + 9}$

$= \frac{-2 - 11i}{25}$

$= \frac{-2}{25} - \frac{11i}{25}$

4. $\frac{3 + \sqrt{-2}}{4 - \sqrt{-2}} = \frac{3 + i\sqrt{2}}{4 - i\sqrt{2}}$

$= \frac{3 + i\sqrt{2}}{4 - i\sqrt{2}} \cdot \frac{4 + i\sqrt{2}}{4 + i\sqrt{2}}$

$= \frac{(12 - 2) + (3\sqrt{2} + 4\sqrt{2})i}{16 + 2}$

$= \frac{10 + 7i\sqrt{2}}{18}$

$= \frac{5}{9} + \frac{7i\sqrt{2}}{18}$

ANSWERS

ENRICHMENT 1

1. c
2. d
3. a
4. c

ENRICHMENT 2

x	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-i	1
-i	-i	i	i	-1

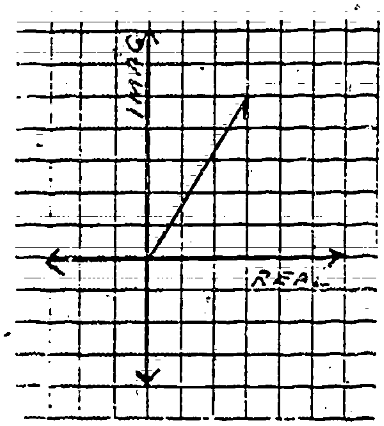
1. Group Properties:

   closure: see chart.
   identity element: 1
   multiplicative inverses:
   1,1; -1,-1; i,-i; -i,i
   associativity: see chart.

2. Commutativity: see chart.

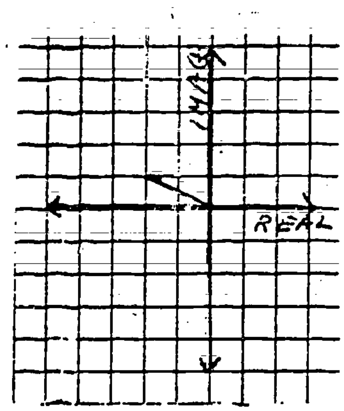
ENRICHMENT 3

1.

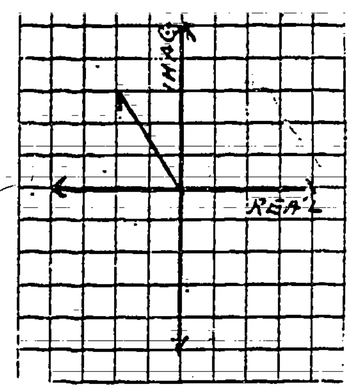


ENRICHMENT 3 (continued)

2.



3.



UNIT III - FACTORING AND SIMPLIFYING RATIONAL EXPRESSIONS

PURPOSE

The purpose of the unit is to reinforce Algebra 1 factoring skills and to extend these skills to a higher order of difficulty.

The unit is prerequisite to Unit VI and all others utilizing these skills.

OVERVIEW

Building upon factoring skills, students will factor polynomial expressions. These skills are next utilized as the students manipulate rational expressions and complex fractions.

SUGGESTIONS TO THE TEACHER

Suggested time frame   The unit is approximately twelve days.

Special attention should be paid to the diagnostic test: students generally enroll in Algebra 2 with very weak factoring skills. The allocated time allows for review of entering performance objectives.

In simplifying the difference of rational expressions, emphasize the distribution of the negative sign.

Factoring the sum of two squares, $a^2 + b^2 = (a + bi)(a - bi)$ is included.

Enrichment is included within certain assessment items.

Computer Applications: Algebra Two with Trigonometry, Foster, pp. 284, 296.

VOCABULARY

binomial
complex fraction
constant term
linear term
monomial
polynomial
quadratic term
rational expression
term
trinomial

ENTERING PERFORMANCE OBJECTIVES

1. Write the factors of an expression containing a common monomial factor.
2. State the factors of an expression th the sum or difference of two perfect squares.
3. Write the factors of a perfect square trinomial expression.
4. State the factors of a trinomial expression as the produce of two binomials.
5. Given a polynomial expression containing four monomial terms, write an equivalent expression that is the product of two binomials.
6. Write the simplest form of the quotient of two monomials in one or more variables.
7. Determine the degree of a monomial.
8. Determine the degree of a polynomial.

DIAGNOSTIC TEST KEYED TO ENTERING PERFORMANCE OBJECTIVES

1. Factor each of the following expressions:

    a) $a^2 - 2a$ a) ______________

    b) $3x^2 - 6xy$ b) ______________

    c) $5y - 25y^2$ c) ______________

2. Factor each expression into a product of two binomials:

    a) $m^2 - n^2$ a) ______________

    b) $a^2 - 3$ b) ______________

    c) $81 + x^2$ c) ______________

3. Factor each expression into a product of two binomials:

    a) $x^2 + 6x + 9$ a) ______________

    b) $x^2 - 8x + 16$ b) ______________

    c) $4x^2 - 4x + 1$ c) ______________

4. Factor each expression into a product of two binomials:

    a) $x^2 - 2x - 3$ a) ______________

    b) $2x^2 + 11x + 5$ b) ______________

    c) $6x^2 + 13x + 6$ c) ______________

5. Use the distributive property to factor each expression into the product of two binomials:

    a) $cx - xd + cy - yd$ a) ______________

    b) $6ax - 4bx + 2by - 3ay$ b) ______________

6. Write each of the following expressions in simplest form:

a) $\frac{2a^2b}{ab}$ a) ______

b) $\frac{-9xy^3z^2}{-3y^4z}$ b) ______

c) $\frac{810a^5b^4c^9d^2}{-1620a^5dbc}$ c) ______

7. Determine the degree of each of the following:

a) $7x^2$ a) ______

b) -4 b) ______

c) $9x^2y^3$ c) ______

8. Determine the degree of each of the following:

a) $3x^2 - 2x + 4$ a) ______

b) $6x^2y - 4xy - 7x - 2$ b) ______

c) $4x^2y^3 - 2x^3y^2$ c) ______

DIAGNOSTIC TEST

ANSWERS

1. a) $a(a-2)$
   b) $3x(x - 2y)$
   c) $5y(1 - 5y)$

2. a) $(m - n)(m + n)$
   b) $(a - \sqrt{3})(a + \sqrt{3})$
   c) $(9 + xi)(9 - xi)$

3. a) $(x + 3)(x + 3)$
   b) $(x - 4)(x - 4)$
   c) $(2x - 1)(2x - 1)$

4. a) $(x - 3)(x + 1)$
   b) $(2x + 1)(x + 5)$
   c) $(2x + 3)(3x + 2)$

5. a) $(c - d)(x + y)$
   b) $(3a - 2b)(2x - y)$

6. a) $2a$
   b) $\frac{3xz}{y}$
   c) $\frac{-b^3c^6d}{2}$

7. a) degree 2
   b) degree 0
   d) degree 5

8. a) degree 2
   b) degree 3
   c) degree 5

UNIT III - FACTORING AND SIMPLIFYING RATIONAL EXPRESSIONS

PERFORMANCE OBJECTIVES

1. Factor a sum of two cubes.
2. Factor a difference of two cubes.
3. Factor completely, over the set of complex numbers, a difference of two terms in the form $a^m - b^n$, where $m > 3$, $n > 4$, and m and n are multiples of 2 or 3.
4. Factor completely a given polynomial expression.
5. State the simplest form of a rational expression by factoring the numerator and denominator over the set of integers.
6. Write a product or quotient of expressions as a single rational expression in simplest form.
7. Write a sum or difference of rational expressions as an equivalent rational expression in simplest form.
8. Given a complex fraction, write it in simplest form.

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (19[illegible])	Sorgenfrey (1973)	Travers (1978)
1	133-135	187	183-184	113-114	198-199	67-68	72 77	177-197	99-101
2	133-135	187	183-184	113-114	198-199	67-68	72 77	177-197	99-101
3	135	188	183-184	--	200-201	57-58 69	75-77	179	--
4	134-135	188	183-184	116-121	200-201	51-52	77	178-179	99-101
5	164-167	196	192-19[illegible]	[illegible]78-2[illegible]0	260-262	72-74	327-328	217-220	125-127
6	167-170	201-202	197[illegible]	[illegible]78-284	260-263	75-80	328-[illegible]30	221-223	128-130
7	170-173	204-205	199-201	286-289	266-271	81-87	330-333	224-227	131-133
8	173-175	204-205	199-201	282-284	272-274	87-88	--	227-229	135-137

III-17

PERFORMANCE OBJECTIVE III-1

Factor a sum of two cubes.

1. Factor:

   $x^3 + 27$

2. Factor:

   $8x^3 + 1$

3. Factor:

   $x^3 + 27y^3$

4. Factor:

   $27x^3 + 64y^3$

PERFORMANCE OBJECTIVE III-2

Factor a difference of two cubes.

1. Factor:

   $x^3 - 1$

2. Factor:

   $8x^3 - 27$

3. Factor:

   $x^3 - 8y^3$

4. Factor:

   $125x^3 - 27y^3$

PERFORMANCE OBJECTIVE III-3

Factor completely, over the set of complex numbers, a difference of two terms in the form $a^m - b^n$, where $m > 3$, $n > 3$, and m and n are multiples of 2 or 3.

1. Factor completely:

   $x^4 - 16$

2. Factor completely:

   $x^4 - y^4$

3. Factor completely:

   $2x^6 - 128$

4. Factor completely:

   $16x^4 - y^{12}$

5. Factor completely:

   $x^{18} - y^{36}$

6. Factor completely:

   $2x^4 - 98$

PERFORMANCE OBJECTIVE III-4

Factor completely a given polynomial expression.

1. Factor completely, using the distributive property:

   $xy + 2x - 3y - 6$

2. Factor completely, using the distributive property:

   $ac + bc + ad + bd$

3. Factor completely, using the distributive property:

   $ax^2 + 3by + bxy + 3ax$

4. Factor completely, using the distributive property:

   $2acxy + bd + 2bcy + axd$

5. Factor completely:

   $(a - b)^5 + 4(b - a)^3$

6. Factor completely:

   $r^4 + r^2s^2 + s^4$

7. Factor completely:

   $5x^2 + 2\sqrt{30}x + 6$

PERFORMANCE OBJECTIVE III-5

State the simplest form of a rational expression by factoring the numerator and denominator over the set of integers.

1. Factor and state in simplest form:

$$\frac{x^2 - 2x - 8}{xy + 2y + 3x + 6}$$

2. Factor and state in simplest form:

$$\frac{x^3 - 1}{x^2 + x + 1}$$

3. Factor and state in simplest form:

$$\frac{2x^3y + 16y}{x + 2}$$

4. Factor and state in simplest form:

$$\frac{x^2 + 10x + 25}{x^3 + 125}$$

PERFORMANCE OBJECTIVE III- 6

Write a product or quotient of expressions as a single rational expression in simplest form.

1. Write as a single rational expression in simplest form:

$$\frac{3ab^2 - 3a}{x^2 - 5x + 6} \cdot \frac{x^3 - 3x^2}{9ab^3 + 9ab^2}$$

2. Write as a single rational expression in simplest form:

$$\frac{16 - x^2}{2x^2 + 11x + 12} \cdot \frac{4x + 6}{x^2 - 10x + 24}$$

3. Write as a single rational expression in simplest form:

$$\frac{12x^2 - 14x - 10}{8x + 4} \div \frac{3x^2 - 11x + 10}{x^3 - 8}$$

4. Write as a single rational expression in simplest form:

$$\frac{12 + 11x - 5x^2}{4x^2 + 4x} \cdot \frac{6x^2 - x - 7}{x^2 - 9} \div \frac{30x^2 - 11x - 28}{4x^2 + 12x}$$

PERFORMANCE OBJECTIVE III-7

Write a sum or difference of rational expressions as an equivalent rational expression in simplest form.

1. Write an equivalent rational expression in simplest form:

$$\frac{1}{a - b} + \frac{a + b}{a^2 + ab + b^2}$$

2. Write an equivalent rational expression in simplest form:

$$\frac{5}{x^3 - 3} + \frac{4}{3 - x^3}$$

3. Write an equivalent rational expression in simplest form:

$$\frac{1}{2 - x} + \frac{2x^2 - 3x - 2}{(x - 2)^3} - \frac{x + 1}{(x - 2)^2}$$

4. Write an equivalent rational expression in simplest form:

$$\frac{a - b}{a^2 - ab + b^2} + \frac{b^2}{a^3 + b^3} - \frac{1}{a + b}$$

PERFORMANCE OBJECTIVE III-8

Given a complex fraction, write it in simplest form.

1. Write in simplest form:

$$\frac{x+1}{1+\frac{1}{x}}$$

2. Write in simplest form:

$$\frac{\frac{2}{3}+\frac{1}{4}+\frac{x}{5}}{\frac{x}{3}+\frac{3}{4}}$$

3. Write in simplest form:

$$\frac{\frac{3}{x^2+4x+4}}{\frac{x-2}{3x^2-12}}$$

4. Write in simplest form:

$$\frac{a+3+\frac{7}{a-2}}{a+4+\frac{5}{a-2}}$$

ANSWERS

III-1

1. $(x + 3)(x^2 - 3x + 9)$
2. $(2x + 1)(4x^2 - 2x + 1)$
3. $(x + 3y)(x^2 - 3xy + 9y^2)$
4. $(3x + 4y)(9x^2 - 12xy + 16y^2)$

III-2

1. $(x - 1)(x^2 + x + 1)$
2. $(2x - 3)(4x^2 + 6x + 9)$
3. $(x - 2y)(x^2 + 2xy + 4y^2)$
4. $(5x - 3y)(25x^2 + 15xy + 9y^2)$

III-3

1. $(x - 2)(x + 2)(x + 2i)(x - 2i)$
2. $(x - y)(x + y)(x + yi)(x - yi)$
3. $2(x - 2)(x + 2)(x^2 + 2x + 4)(x^2 - 2x + 4)$
4. $(2x - y^3)(2x + y^3)(2x + iy^3)(2x - iy^3)$
5. $(x + y^2)(x - y^2)(x^2 + xy^2 + y^4)(x^2 - xy^2 + y^4)(x^6 + x^3y^6 + y^{12})$ $(x^6 - x^3y^6 + y^{12})$
6. $2(x - \sqrt{7})(x + \sqrt{7})(x - i\sqrt{7})$

ANSWERS (continued)

III-4

1. $(x + [illegible])(y + 2)$

2. $([illegible] + b)(c + d)$

3. $(x + 3)\ (ax + by)$

4. $(a[illegible] - b)(2cy + [illegible])$

5. $(a - b)^3(a - b + 2)(a - b - 2)$

6. $(r^2 + s^2 - rs)(r^2 + s^2 + rs)$

7. $(\sqrt{5}x + \sqrt{6})^2$

III-5

1. $\frac{(x - 4)(x + 2)}{(x + 2)(y + 3)} = \frac{x - 4}{y + 3}$

2. $\frac{(x - 1)(x^2 + x + 1)}{x^2 + x + 1} = x - 1$

3. $\frac{2y(x^3 + 8)}{x + 2} = \frac{2y(x + 2)\ (x^2 - 2x + 4)}{x + 2} = 2y(x^2 - 2x + 4)$

4. $\frac{(x + 5)(x + 5)}{(x + 5)(x^2 - 5x + 25} = \frac{x + 5}{(x^2 - 5x + 25)}$

III-6

1. $\frac{3a(b - 1)(b + 1)}{(x - 2)(x - 3)} \bullet \frac{x^2(x - 3)}{9ab^2(b + 1} = \frac{x^2(b - 1)}{3b^2(x - 2)}$

2. $\frac{(4 - x)(4 + x)}{(2x - 3()x + 4)} \bullet \frac{2(2x + 3)}{(x - 4)(x - 6)} = \frac{-2}{x - 6}$ or $\frac{-2}{x - 6}$ or $\frac{2}{6 - x}$

3. $\frac{2(2x + 1)(3x - 5)}{4(2x + 1)} \div \frac{(x - 2)(3x - 5)}{(x - 2)(x^2 + 2x + 4)}$

$= \frac{2(2x + 1)(3x - 5)}{4(2x + 1)} \bullet \frac{(x - 2)(x^2 + 2x + 4)}{(x - 2)(3x - 5)} = \frac{x^2 + 2x + 4}{2}$

4. $\frac{(3 - x)(4 + 5x)}{4x(x + 1)} \bullet \frac{(x + 1)(6x - 7)}{(x - 3)(x + 3)} \div \frac{(5x + 4)(6x - 7)}{4x(x + 3)}$

$= \frac{(3 - x)(4 + 5x)}{4x(x + 1)} \bullet \frac{(x + 1)(6x - 7)}{(x - 3)(x + 3)} \circ \frac{4x(x + 3)}{(5x + 4)(6x - 7)} = \frac{3 - x}{x - 3} = -1$

ANSWERS (continued)

III-7

1. $\frac{2a^2 + ab}{a^3 - b^3}$ or $\frac{a(2a + b)}{a^3 - b^3}$

2. $\frac{1}{x^3 - 3}$ or $\frac{-1}{3 - x^3}$

3. $\frac{2}{(x - 2)^2}$

4. $\frac{ab - b^2}{a^3 + b^3}$ or $\frac{b(a - b)}{a^3 + b^3}$

III-8

1. x

2. $\frac{12x + 55}{20x + 45}$

3. $\frac{9}{x + 2}$

4. $\frac{a^2 + a + 1}{a^2 + 2a - 3}$

UNIT IV - RELATIONS AND FUNCTIONS

PURPOSE

Functions are generally considered the unifying concept of mathematics. It is essential that students be able to distinguish a function (and its properties) from among various relations. Further, an understanding of functions is prerequisite to the study of second-degree equations.

OVERVIEW

Although introduced in Algebra 1, functions are rigorously developed in this unit. The analysis of graphs of relations and functions is reviewed and extended. The absolute value concept and its inclusion in functions is studied in detail.

SUGGESTIONS TO THE TEACHER

Teachers should emphasize the use of functional notation throughout the unit (e.g., $y = f(x)$).

It is essential that students planning to pursue advanced programs in mathematics developed the analytical techniques associated with certain unit objectives.

The use of composite function notation varies with the instructional material being used. Generally, $f(g(x))$ is synonymous with $f \circ g(x)$ or $f \circ g$. A variety of exercises using this concept is recommended for students planning to take other mathematics courses.

The teachers should emphasize, with several examples, the distinction between an equation and an expression.

The entering performance objectives for the unit may require extended emphasis. The suggested time of 20 days for the unit provides for this extended treatment.

Computer Applications: Algebra Two with Trigonometry, Foster, p. 269.

VOCABULARY

relation	linear equation
function	absolute value
intercept	coordinate
composition	domain
abscissa	range
ordinate	constant function

UNIT IV - RELATIONS AND FUNCTIONS

ENTERING PERFORMANCE OBJECTIVES

1. Select equations of the form $ax + by + c = 0$ from among a list of equations of various types.

2. Write two ordered pairs of real numbers contained in the solution set of a given equation of the form $ax + by + c = 0$.

3. Write the abscissa and ordinate for a given ordered pair of real numbers.

4. Construct a table containing at least three ordered pairs and construct the graph for a given linear equation (of the form $ax + by + c = 0$).

5. State a definition of an: a) x-intercept
   b) y-intercept

6. State the intercepts, given a linear equation (of the form $ax + by + c = 0$).

7. Write the slope of a line given two points.

8. Determine the slope and y-intercept of a linear equation by transforming the equation to the form $y = mx + b$.

9. Write an equation of a line, given the slope and a point of the line.

10. Write the equation of a line in the form $ax + by + c = 0$, given any two points of the line.

11. Construct the graph of a linear inequality.

UNIT IV - FUNCTIONS AND RELATIONS

DIAGNOSTIC TEST KEYED TO ENTERING PERFORMANCE OBJECTIVES

1. Select from the following list the equations which are in the form $ax + by + c = 0$.

   a) $y + 1 = 0$

   b) $-2x + 3y + 4 = 0$

   c) $4xy - 2 = 0$

   d) $x^2 + 2y^2 + 1 = 0$

   e) $x + 4 = 0$

   ____

2. Write two ordered pairs of numbers that satisfy the equation:

   $\{(x, y): 2x + y = 6\}$

   a) ____________________

   b) ____________________

3. Write a) the abscissa and b) the ordinate for the ordered pair $(-4, \frac{1}{2})$.

   a) ____________________

   b) ____________________

4. Construct a table of values and the graph of the equation:

   $\{(x, y) : -x + y = 4\}$

5. State a definition of:

   a) x-intercept

   b) y-intercept

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6. For the equation $\{(x, y) : 2x - 5y = 30\}$, state the x- and y-intercepts.

x-intercept ______________

y-intercept ______________

7. State the slope of a line that passes through the points $P_1(-5, 4)$ and $P_2(1, -2)$. ______________

8. Transform the equation $\{(x, y) : \frac{1}{3}x + 2y - 1 = 0\}$ to the slope-intercept form. State the slope and y-intercept of the open sentence.

Equation:

Slope ______________

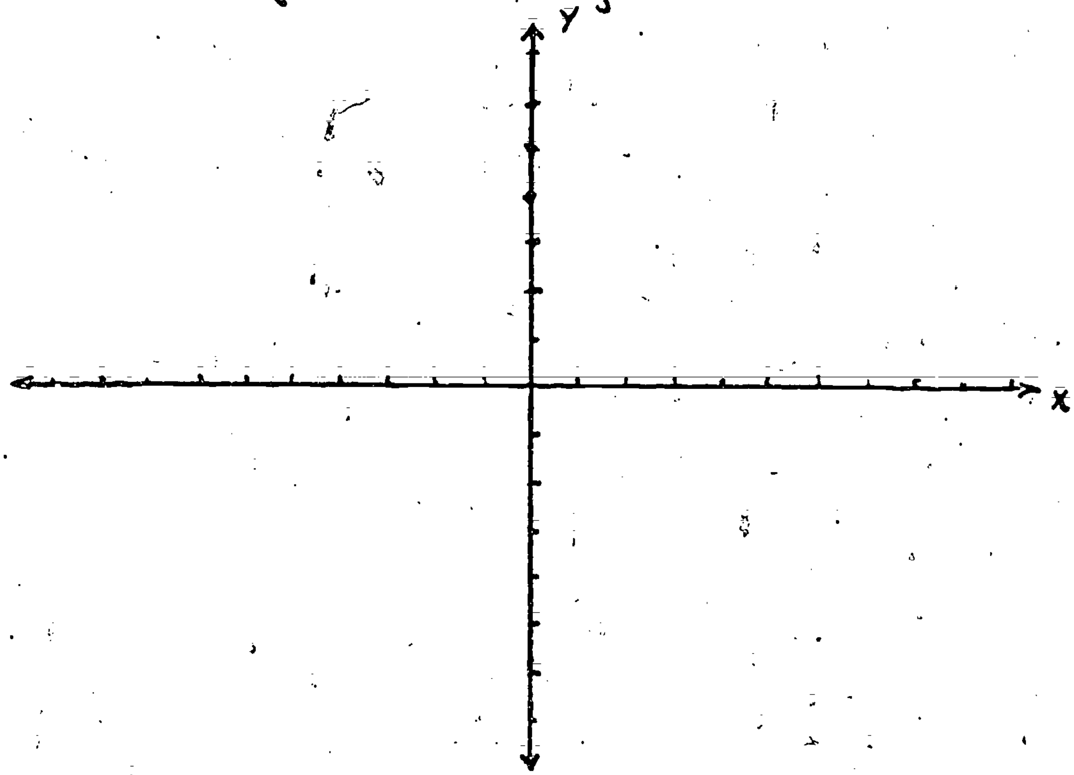
y-intercept ______________

9. Write the equation of the line that contains the points $P_1(3,5)$ and whose slope is 2.

____

10. Write the equation of a line, in the form $ax + by + c = 0$, which contains the points $P_1(3, 5)$ and $P_2(6, -1)$.

11. Construct the graph of $\{(x, y) : 2x + y \leq 1\}$



DIAGNOSTIC TEST

ANSWERS

1. a, b, and e

2. a) (2,2) Answers will vary.

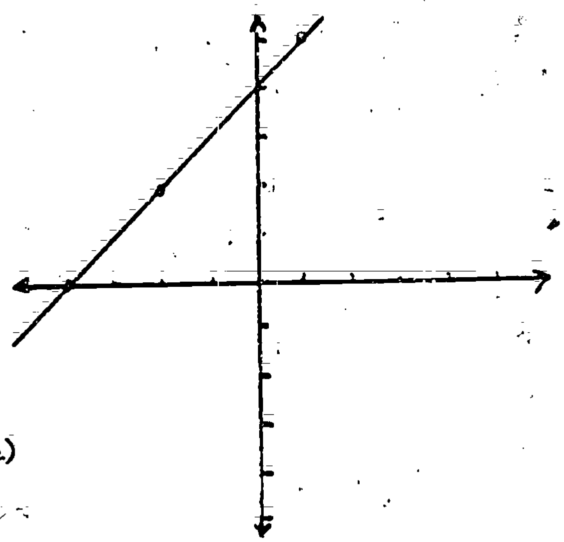
   b) (3,0)

3. a) -4

   b) $\frac{1}{2}$

4.

x	y
1	5
-4	0
-2	2



(Other answers are possible)

5. Acceptable answers include:

   a) where a line crosses the x-axis; where the y-coordinate is zero

   b) where a line crosses the y-axis; where the x-coordinate is zero

6. x-intercept : 15

   y-intercept : -6

7. -1

8. Suggested acceptable demonstration:

$$-\frac{1}{3}x + 2y - 1 = 0$$

$$2y = \frac{1}{3}x + 1$$

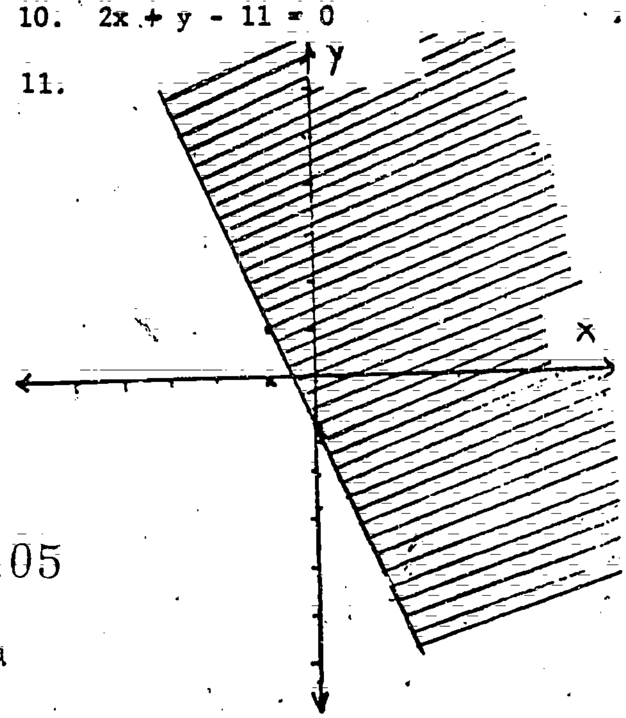
$$\boxed{y = \frac{1}{6}x + \frac{1}{2}}$$

Slope is $\frac{1}{6}$, y-intercept is $\frac{1}{2}$

9. $y = 2x + 8$

10. $2x + y - 11 = 0$

11.



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UNIT IV - RELATIONS AND FUNCTIONS

PERFORMANCE OBJECTIVES

1. State the domain and range of a relation.

2. Distinguish between a function and a relation that is not a function, given several relations defined by sets of ordered pairs.

3. Determine whether a relation is a function, given the graph of the relation.

4. Determine the value of an algebraic function, given a value of the domain.

5. Construct the graph of a linear function in the form $f(x) = mx + b$.

6. Construct the graph of a constant function.

7. Demonstrate the procedure for finding the algebraic rule of a composite function, given the rule for each subfunction.

8. Determine the solution set of, and construct the graph for, a linear equality in one variable that contains an absolute value expression.

9. Determine the solution set of, and construct the graph for, a linear inequality in one variable that contains an absolute value expression.

10. Construct the graph of a linear function defined by a rule involving absolute value.

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	209-212	69-71	67-69	41	216-217	180-183	92-95	131-134	224-229
2	209-215	73	70-73	41-43	--	183-186	95-96	131-137	230
3	209-215	74	70-73	41-45	231 234-235	184-186	--	131-137	230
4	213-215	70-71	74-76	41-45	233-236	186	97	135-137	232-234
5	84-95	76-79	74-76	46-48	248 250	186	98-103	138-143	235-238
6	213-217	91	74-76	46-48	--	190 193	121-122 124	138-143	235-237
7	216	71	--	--	251	186. 197-200	97-98	138	--
8	15, 18 52-53	60-63	58-61	--	139-142	112-114	63-66	39-43	53-54 253
9	52-56	60-63	58-61	--	140-142	127-128	64-66	39-43 65-67	53-54
10	209-212	78	58-61	56-58	147 219 241-243	159-161 193	153	131-134	249 253-256

PERFORMANCE OBJECTIVE IV-1

State the domain and range of a relation.

1. State the domain and range of the following relation:

$\{(0, 1), (4, -2), (5, -3), (5, 2), (-5, 3)\}$

Domain: ______________

Range: ______________

2. State the domain and range of the following relation:

$\{(3, 4), (5, -4), (7, 1), (8, -1), (3, 3)\}$

Domain: ______________

Range: ______________

3. State the domain and range of the following relation:

$\{(-2, 2), (-1, 0), (0, 0), (4, 4)\}$

Domain: ______________

Range: ______________

4. State the domain and range of the following relation:

$\{(10, 1), (20, 2), (30, 3), (30, 2)(20, 1)\}$

Domain: ______________

Range: ______________

PERFORMANCE OBJECTIVE IV-2

Distinguish between a function and a relation that is not a function, given several relations defined by sets of ordered pairs.

1. Each of the following sets defines a relation. Write the letter of each relation that defines a function.

   a) $\{(0, 1), (1, 2), (-1, 3.5)\}$

   b) $\{(4, -2), (\sqrt{2}, -2), (1, 1), (4, 3.5)\}$

   c) $\{(0, 0), (5, 0), (-\sqrt{3}, 0), (100, 0)\}$

   d) $\{(2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

   ______

2. Write the letter of each relation that defines a function.

   a) $\{(-1, 1), (-2, 2), (-3, 3), (-4, 4)\}$

   b) $\{(-5, 1), (-4, 8), (-3, 12), (8, -3), (12, -5)\}$

   c) $\{(-1, 0), (-2, 1), (-3, 2), (-2, 3), (-1, 4)\}$

   d) $\{(-3, 4), (20, 1), (15, 0)\}$

   ______

3. Write the letter of each relation that defines a function.

   a) $\{(0, -81), (0, 81), (5, 21), (-5, -21)\}$

   b) $\{(\sqrt{5}, 4), (-\sqrt{5}, -4)\}$

   c) $\{(1, 2), (0, 4), (-2, -1), (0, 0)\}$

   d) $\{(-\sqrt{3}, -\sqrt{3}), (\sqrt{3}, \sqrt{3}), (\sqrt{5}, -\sqrt{5}), (2\sqrt{3}, \sqrt{5})\}$

   ______

4. Write the letter of each relation that defines a function.

a) $\{(10, -2), (-9, 5), (4, 3), (8, 6)\}$

b) $\{(7, 5), (\frac{1}{2}, \frac{1}{4}), (\frac{2}{3}, \frac{1}{5})\}$

c) $\{(-\frac{1}{2}, 2), (-\frac{1}{3}, 3), (\frac{1}{5}, -5), (\frac{1}{4} - 4)\}$

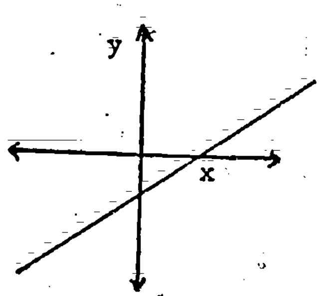
d) $\{(-1, -2), (-2, -3), (0, 0), (-2, -2), (-2, 3)\}$

____

PERFORMANCE OBJECTIVE IV-3

Determine whether a relation is a function, given the graph of the relation.

1. Pictured below is the graph of a relation. Determine whether or not the relation is a function with respect to x.



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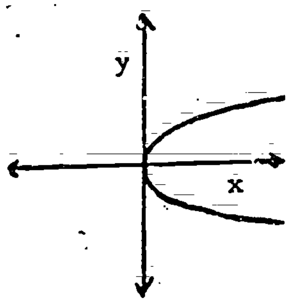
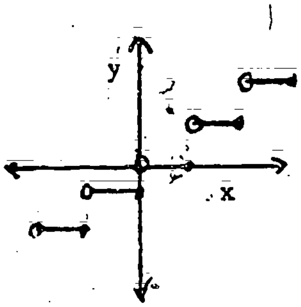
PERFORMANCE OBJECTIVE IV-3 (continued)

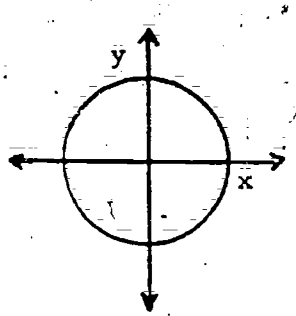
2. Pictured below is the graph of a relation. Determine whether or not the relation is a function with respect to x.



3. Pictured below is the graph of a relation. Determine whether or not the relation is a function with respect to x.



4. Pictured below is a graph of a relation. Determine whether or not the relation is a function with respect to x.



PERFORMANCE OBJECTIVE IV-4

Determine the value of an algebraic function, given a value in the domain.

1. Determine the value of the function $f(x) = 2x - 5$ when $x = -3$.

2. Determine the value of the function $g(x) = x^2 - 4$ when $x = -2$.

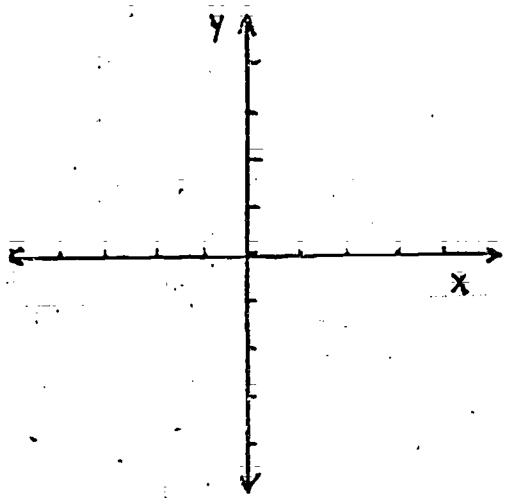
3. Determine the value of the function $m(x) = -5x - 2$ when $x = 0$.

4. Determine the value of the function $h(x) = 2x^3 - 2x + 1$ when $x = 1$.

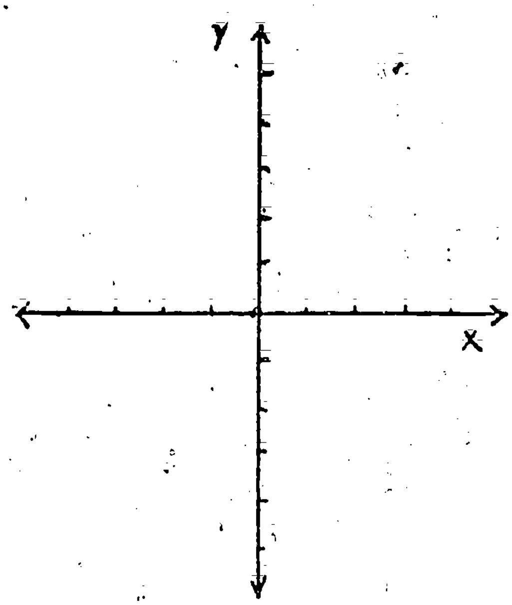
PERFORMANCE OBJECTIVE IV-5

Construct the graph of a linear function in the form $f(x) = mx + b$.

1. Construct the graph of the linear function $f(x) = -2x + 1$.

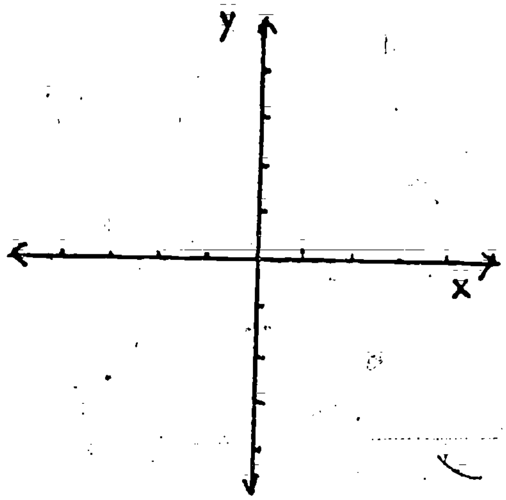


2. Construct the graph of the linear function $f(x) = 3x - 2$.

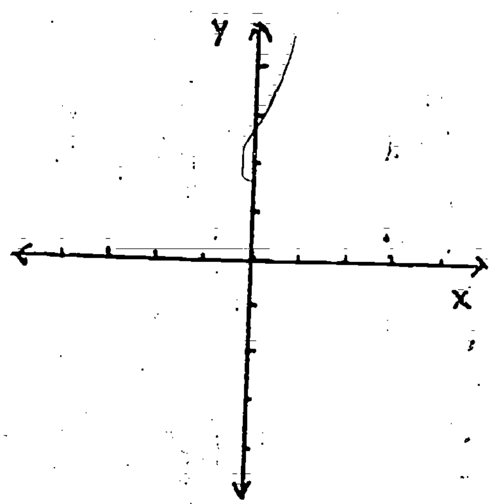


PERFORMANCE OBJECTIVE IV-5 (continued)

3. Construct the graph of the linear function $f(x) = -x - 5$.



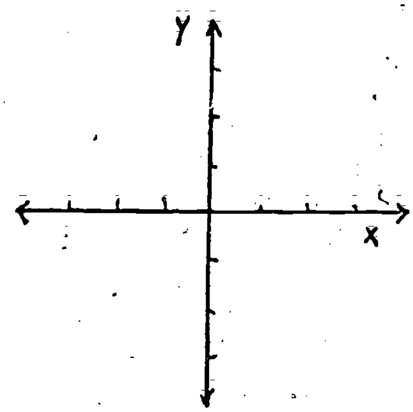
4. Construct the graph of the linear function $f(x) = 5x + 3$.



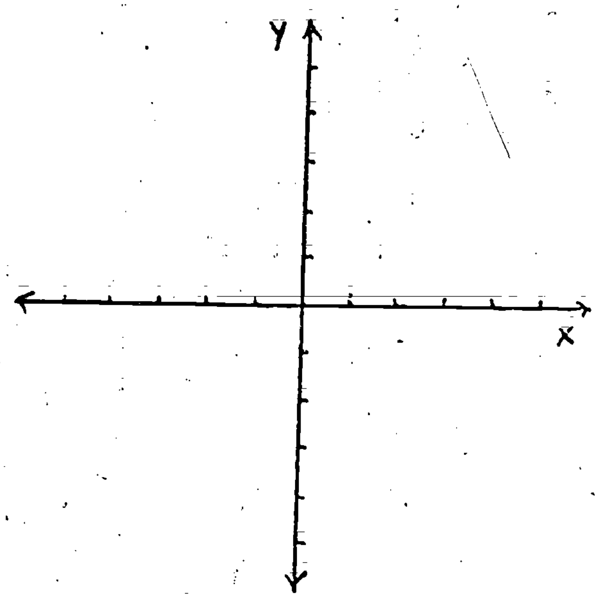
PERFORMANCE OBJECTIVE IV-6

Construct the graph of a constant function.

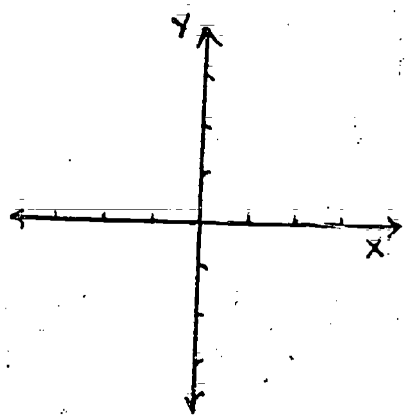
1. Construct the graph of $f(x) = 2$.



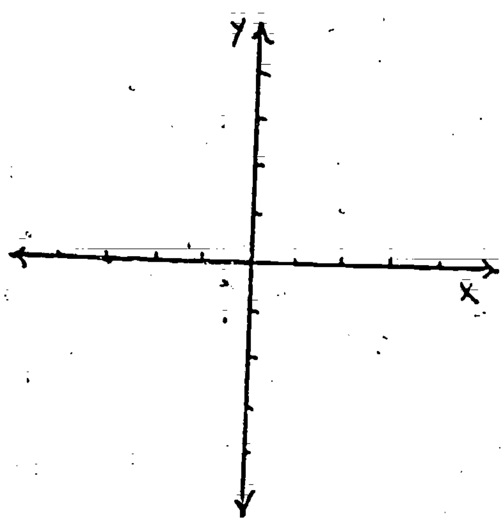
2. Construct the graph of $h(x) = -4$.



3. Construct the graph of $g(x) = 1$.



4. Construct the graph of $f(x) = -3$.



PERFORMANCE OBJECTIVE IV-7

Demonstrate the procedure for finding the algebraic rule of a composite function, given the rule for each subfunction.

1. Demonstrate the procedure for finding $g(f(x))$, where $f(x) = 3x$ and $g(x) = 2x + 1$. Write the answer in simplest form.

2. Demonstrate the procedure for finding $g(f(x))$, where $f(x) = x - 1$ and $g(x) = x^2 - 1$. Write the answer in simplest form.

PERFORMANCE OBJECTIVE IV-7 (continued)

3. Demonstrate the procedure for finding $g(f(x))$, where $f(x) = 2\sqrt{x}$ and $g(x) = 3x^2 + 1$. Write the answer in simplest form.

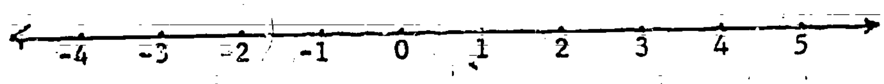
4. Demonstrate the procedure for finding $g(f(x))$, where $f(x) = \frac{1}{x + 1}$ and $g(x) = x^3 + x + 1$. Write the answer in simplest form.

PERFORMANCE OBJECTIVE IV-8

Determine the solution set of, and construct the graph for, a linear equality in one variable that contains an absolute value expression.

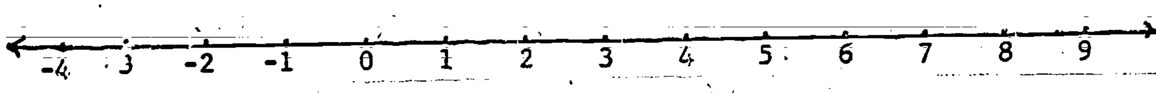
1. Determine the solution set and construct the graph for $|2x| + 1 = 7$.

____



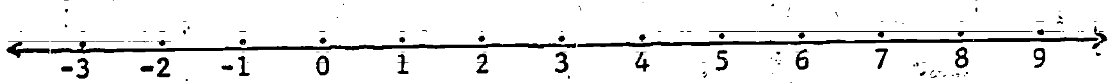
2. Determine the solution set and construct the graph for $|2x - 5| = 7$.

____



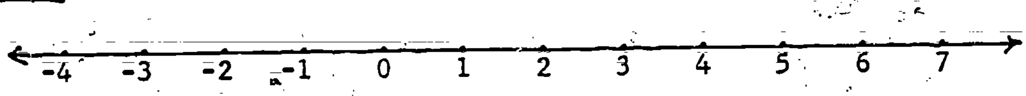
3. Determine the solution set and construct the graph for $|2x + 4| = 3x - 5$.

____



4. Determine the solution set and construct the graph for $|x + 6| = |2x|$.

____

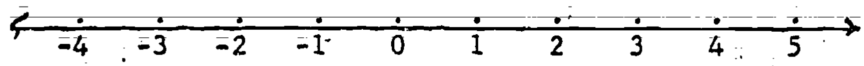


PERFORMANCE OBJECTIVE IV-9

> Determine the solution set of, and construct the graph for, a linear inequality in one variable that contains an absolute value expression.

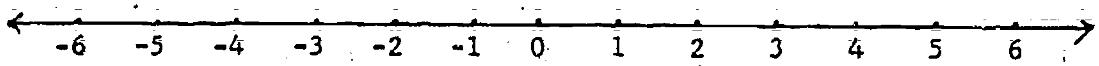
1. Determine the solution set and construct the graph for $|x| < 4$.

______



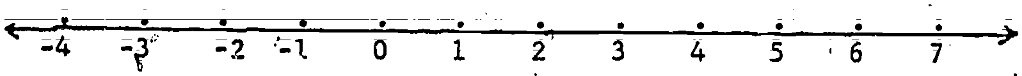
2. Determine the solution set and construct the graph for $|x| \geq 6$.

______



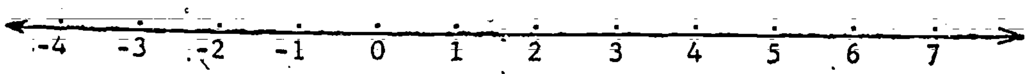
3. Determine the solution set and construct the graph for $|3x + 2| \leq 5$.

______



4. Determine the solution set and construct the graph for $|(2x - 1)| \geq 3$

______



PERFORMANCE OBJECTIVE IV-10

> Construct the graph of a linear function defined by a rule involving absolute value.

1. Construct the graph of $f(x) = |x| + 2$.

2. Construct the graph of $f(x) = 2|x| - 1$.

3. Construct the graph of $f(x) = |x - 3|$.

4. Construct the graph of $f(x) = -2|x + 1|$.

UNIT IV - RELATIONS AND FUNCTIONS

ANSWERS

IV-1

1. Domain: $\{0, 4, 5, -5\}$; Range: $\{1, -2, -3, 2, 3\}$
2. Domain: $\{3, 5, 7, 8\}$; Range: $\{4, -4, 1, -1, 3\}$
3. Domain: $\{-2, -1, 0, 4\}$; Range: $\{2, 0, 4\}$
4. Domain: $\{10, 20, 30\}$; Range: $\{1, 2, 3\}$

IV-2 - (All answers must be listed for mastery)

1. a, c, and d
2. a, b, and d
3. b and d
4. a, b, and c

IV-3

1. is
2. is not
3. is
4. is not

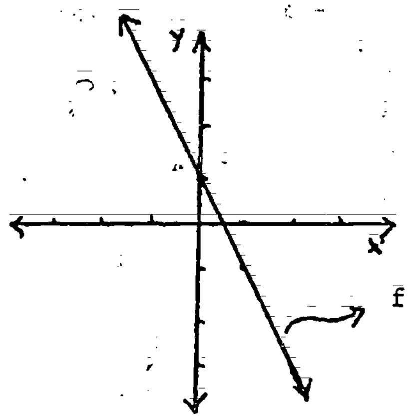
IV-4

1. - 11
2. 0
3. - 2
4. 1

ANSWERS

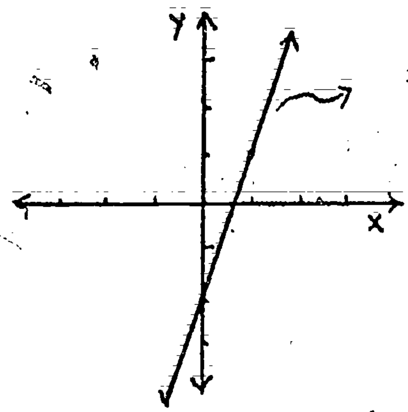
IV-5

1.



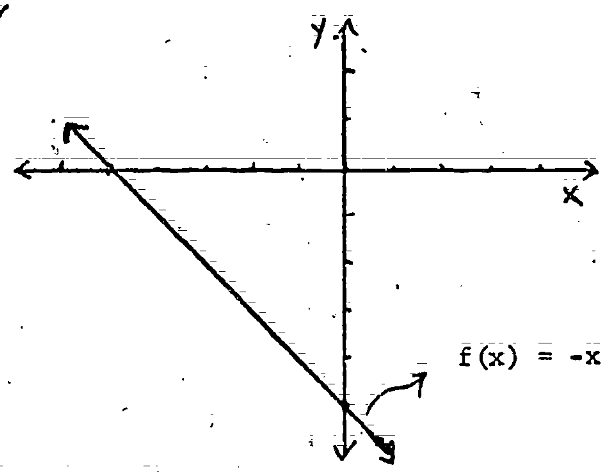
f(x) = -2x + 1

2.



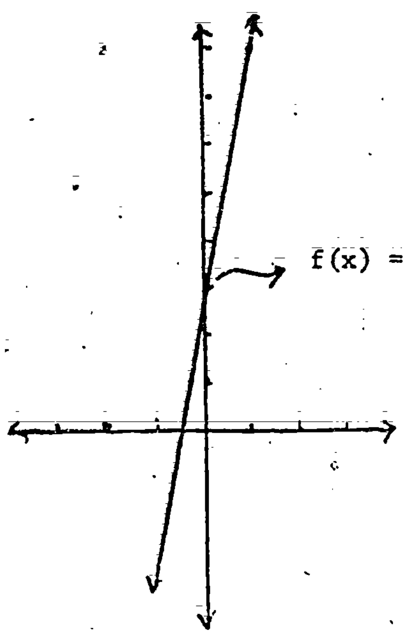
f(x) = 3x - 2

3.



f(x) = -x - 5

4.

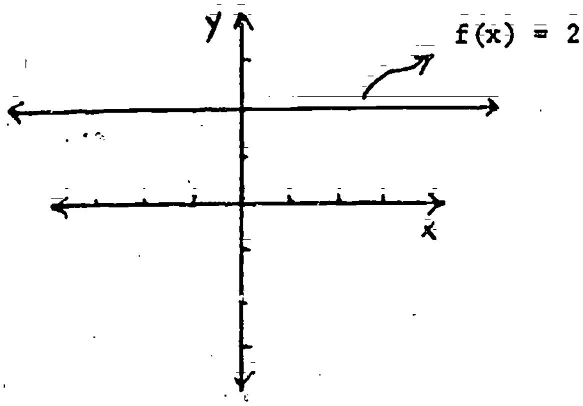


f(x) = 5x + 3

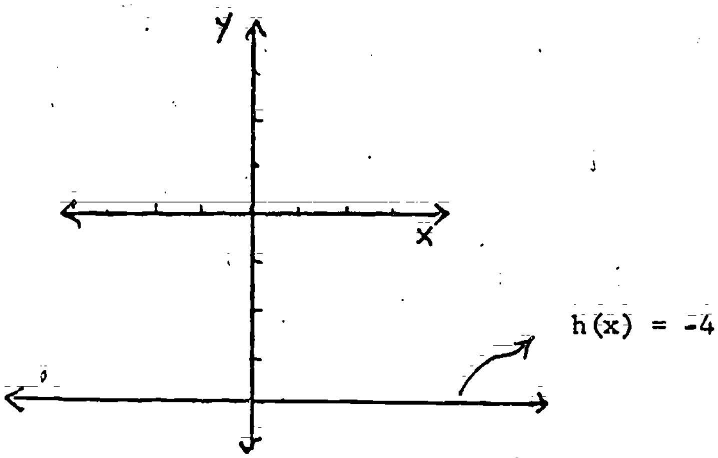
ANSWERS

IV-6

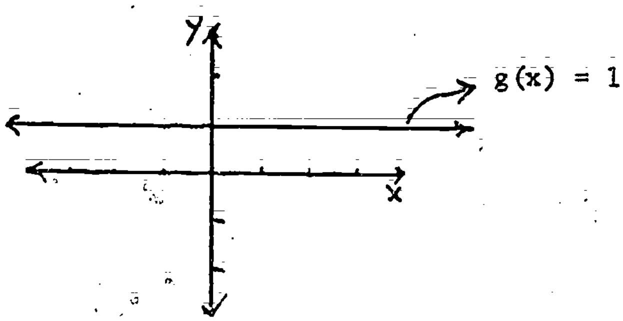
1.



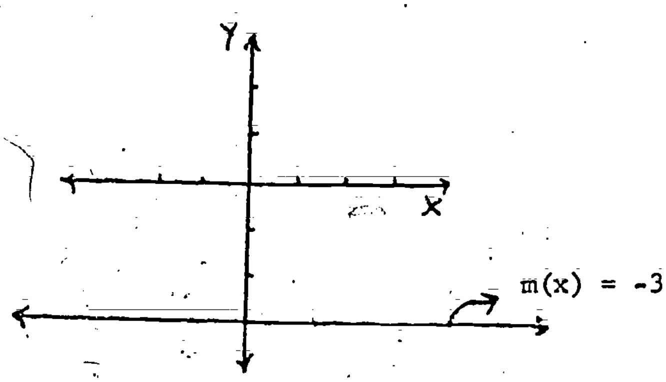
2.



3.



4.



ANSWERS

IV-7

1. $f(x) = 3x$ and $g(x) = 2x + 1$,

   thus $g(f(x)) = 2(3x) + 1$

   $g(f(x)) = 6x + 1$

2. $f(x) = x - 1$ and $g(x) = x^2 - 1$,

   thus $g(f(x)) = (x - 1)^2 - 1$

   $g(f(x)) = x^2 - 2x + 1 - 1$

   $g(f(x)) = x^2 - 2x$ OR $x(x-2)$

3. $f(x) = 2\sqrt{x}$ and $g(x) = 3x^2 + 1$,

   thus $g(f(x)) = 3(2\sqrt{x})^2 + 1$

   $g(f(x)) = 12x + 1$

4. $f(x) = \frac{1}{x + 1}$ and $g(x) = x^2 + x + 1$,

   thus, $g(f(x)) = \left(\frac{1}{x + 1}\right)^2 + \frac{1}{x + 1} + 1$

   $$= \frac{1}{(x + 1)^2} + \frac{1}{x + 1} + 1$$

   $$= \frac{1 + (x + 1) + (x + 1)^2}{(x + 1)^2}$$

   $$= \frac{x^2 + 3x + 3}{(x + 1)^2} \quad \text{OR}$$

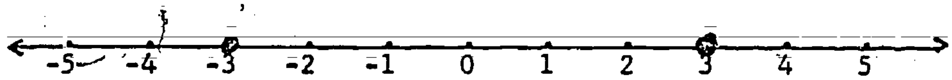
   $$= \frac{x^2 + 3x + 3}{x^2 + 2x + 1}$$

ANSWERS

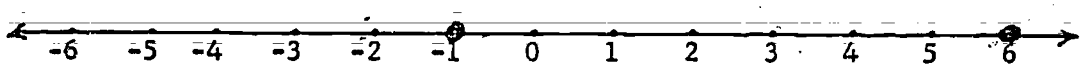
IV-8

(Mastery - both parts correct)

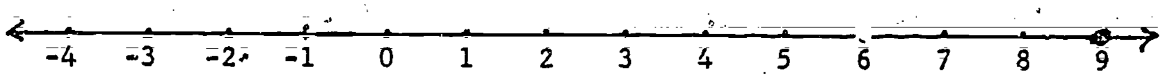
1. $\{3, -3\}$



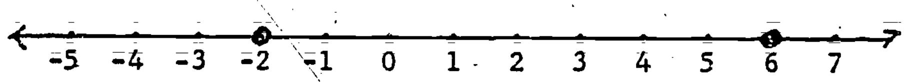
2. $\{6, -1\}$



3. $\{9\}$

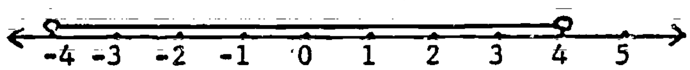


4. $\{6, -2\}$

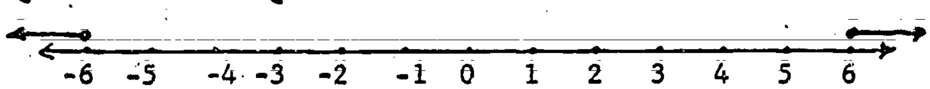


IV-9

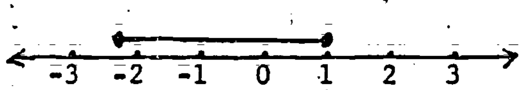
1. $\{x: -4 < x < 4\}$ OR $\{x: x < 4\} \cap \{x: x > -4\}$



2. $\{x: x \geq 6\} \cup \{x: x \leq -6\}$ (Both sets required)



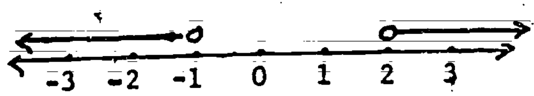
3. $\{x: -\frac{7}{3} \leq x \leq 1\}$ OR $\{x: x \leq 1\} \cap \{x: x \geq -\frac{7}{3}\}$



ANSWERS

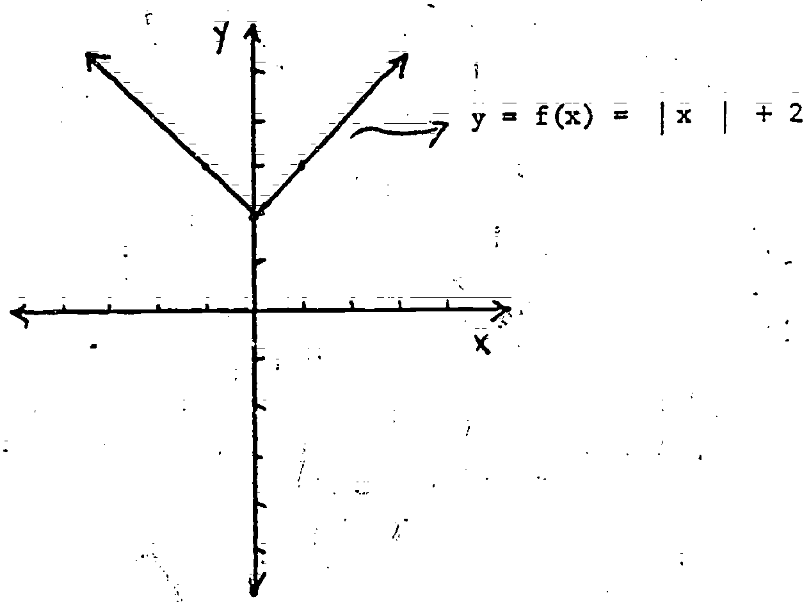
IV-9 (continued)

4. $\{x: x \geq 2\} \cup \{x: x \leq -1\}$

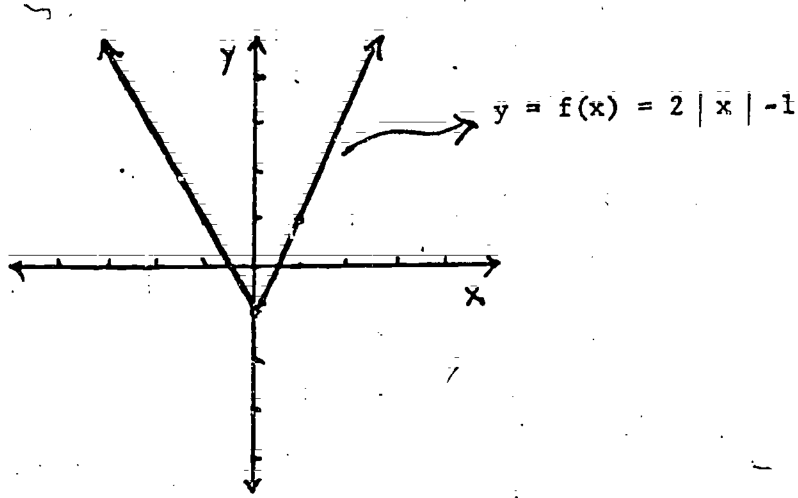


IV-10

1.



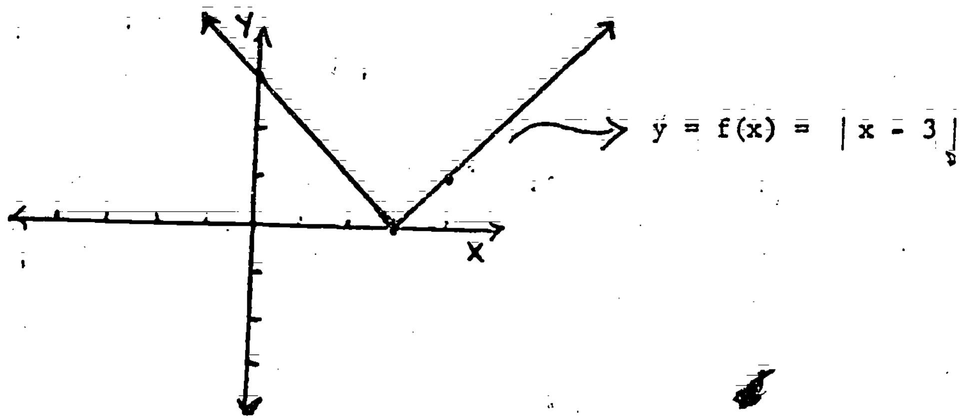
2.



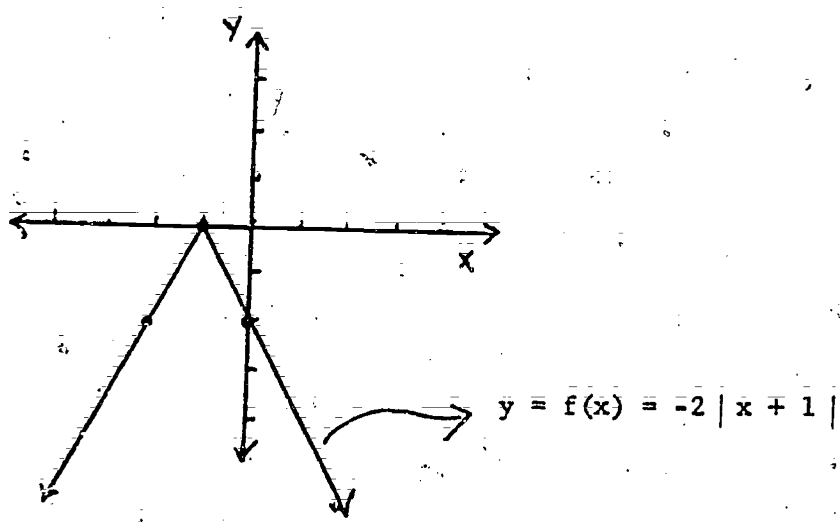
ANSWERS

IV-10 (continued)

3.



4.



UNIT V - POINTS AND FIRST DEGREE EQUATIONS

PURPOSE

This unit is essential to the transition from linear to quadratic topics. It also provides necessary background material for the continued study of mathematics.

OVERVIEW

Distance and midpoint formulas are introduced in preparation for working with parallel and perpendicular lines, isosceles and right triangles. Simple direct variation extends the previous unit's discussion on functions.

SUGGESTIONS TO THE TEACHER

Instead of administering a separate diagnostic test, the teacher may wish to incorporate the entering performance objectives into the lesson. Assessment tasks are given for this purpose. Stating the general equation before working problems in Performance Objectives One and Two may help the students remember the distance and midpoint formulas. The teacher may find that some or all of the material is review for the students; however, the unit is necessary as a transition from linear to quadratic functions. Since the material may be familiar to some students, several higher order assessment tasks are given for some of the objectives. A review of narrative problems from Algebra 1 should be incorporated in this unit.

Computer Applications: Algebra Two and Trigonometry, Keedy, p. 93, "Finding the slope and y-intercept of linear equations"; Algebra 2 and Trigonometry, Dolciani (1978), pp. 101-109.

The time needed to complete the unit is approximately ten days.

VOCABULARY

parallel
perpendicular
collinear
direct variation
proportion
constant of proportionality (variation)
non-collinear
isosceles triangle
right triangle
Pythagorean Theorem
perimeter
distance
midpoint

ENTERING PERFORMANCE OBJECTIVES

1. Use the Pythagorean Theorem to compute the length of any side of a right triangle, given the other two sides.

2. Given the slopes of two lines, identify the conditions which establish that the lines are parallel.

3. Given the slopes of two lines, identify the conditions which establish that the lines are perpendicular.

DIAGNOSTIC TEST KEYED TO ENTERING PERFORMANCE OBJECTIVES

1. Given a right triangle whose legs have lengths 8 and 6 respectively, determine the length of the hypotenuse.

____

2. Given a right triangle whose hypotenuse has length 15 and one leg has length 9, determine the length of the other leg.

____

3. If a and b represent the lengths of the legs of a right triangle, and c the hypotenuse, write an equation showing the relationship among the three measurements.

____

4. Given a right triangle whose sides are represented by $2(x + 1)$ and $2x$, find the hypotenuse.

____

5. Each of the following pairs of numbers represents the slopes of two distinct lines. Select the pair which indicate that the lines are parallel.

a) $-5; 5$ b) $-5; \frac{1}{5}$ c) $-5; -5$ d) $-5; -\frac{1}{5}$

6. If m_1 and m_2 are the slopes of two parallel lines (m_1 and $m_2 \neq 0$) select, from the following, relationships which are true:

a) $m_1 = m_2$

b) $\frac{1}{m_1} = -\frac{1}{m_2}$

c) $\frac{1}{m_1} = -m_2$

d) $m_1 \cdot m_2 = -1$

e) $-\frac{1}{m_1} = -\frac{1}{m_2}$

f) $-m_1 = -m_2$

g) $m_1 = \frac{1}{m_2}$

h) $\frac{1}{m_1} = -(-\frac{1}{m_2})$

i) $\frac{1}{m_1} = \frac{1}{m_2}$

7. Each of the following pairs of numbers represents the slopes of two lines. Select the pair which indicate that the lines are perpendicular.

a) $4; \frac{1}{4}$ b) $4; 4$ c) $4; -\frac{1}{4}$ d) $4; -4$

8. If m_1 and m_2 are the slopes of two perpendicular lines (m_1 and $m_2 \neq 0$), select, from the following, relationships which are true:

a) $m_1 = m_2$

b) $\frac{1}{m_1} = -\frac{1}{m_2}$

c) $\frac{1}{m_1} = -(m_2)$

d) $m_1 \cdot m_2 = -1$

e) $-\frac{1}{m_1} = -\frac{1}{m_2}$

f) $m_1 = -\frac{1}{m_2}$

g) $-\frac{1}{m_1} = m_2$

h) $m_1 = -m_2$

i) $-m_1 = \frac{1}{m_2}$

DIAGNOSTIC TEST

ANSWERS

1. 10

2. 12

3. $a^2 + b^2 = c^2$

4. $2\sqrt{2x^2 + 2x + 1}$

5. c

6. a), e), f), h), i) (at least 4 out of 5 needed for mastery)

7. c)

8. c), d), f), g), i) (at least 4 out of 5 needed for mastery)

UNIT V - POINTS AND FIRST DEGREE EQUATIONS

PERFORMANCE OBJECTIVES

1. Demonstrate the procedure for finding the distance between two points in a Cartesian system, using the distance formula.

2. Demonstrate the procedure for finding the midpoint of a segment, given the endpoints.

3. Given three points in a Cartesian plane, determine if they are collinear.

4. Determine the perimeter of a triangle, given three non-collinear points.

5. Given three non-collinear points, determine if the triangle is isosceles and/or a right triangle.

6. Given an equation of a line, state the slope of the lines parallel and perpendicular to that line.

7. Determine an equation of a line parallel to a given line and passing through a given point.

8. Determine an equation of the line perpendicular to a given line and passing through a given point.

9. Translate a direct variation problem into an equation and solve.

10. Translate a narrative problem involving direct variation into an equation and solve.

11. Translate a narrative problem into an equation and solve.

UNIT V - POINTS AND FIRST DEGREE EQUATIONS

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

Objective	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	299-301	333-336	341-342	214-216	426-427	137-139	273-278	351-353	307-309
2	299-301	334-336	343	215-216	427	140-143	276-278	351-353	307-309
3	299-301 306	--	--	--	--	139	276,278	351-354 358-359	--
4	301,306	335	--	216	89	139	--	354,358	--
5	301	335	344	--	--	139	276,278	354	--
6	303-306	336-338	346	70-72	85-89	149-151	129-133	356-358	--
7	94	89	86-88	70	87-89	152,157	132-133	103	--
8	306	337-338	345-346	71	88-89	152,157	132-133	358	--
9	217-221 230	90-93	89-92	299	289-292	191-193	104-106	143-147 262-264	243-246

UNIT V - POINTS AND FIRST DEGREE EQUATIONS

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

Objective	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
10	222-223 230-232	94	93	297-299	292-293		--	147-148 264-266	243-246
11	--	124-129 209,211	42-44,93 119-124 158-161 202-204	14-17	114-122	119-126	--	70-74 86-87 230-231	38-41

PERFORMANCE OBJECTIVE V-1

Demonstrate the procedure for finding the distance between the points in a Cartesian system, using the distance formula.

1. Demonstrate the procedure for finding the distance between the points $(5, -1)$ and $(7, -1)$.
2. Demonstrate the procedure for finding the distance between the points $(\frac{1}{4}, 3)$ and $(\frac{1}{4}, -7)$.
3. Demonstrate the procedure for finding the distance between the points $(2\sqrt{2}, 5)$ and $(\sqrt{2}, -1)$.
4. Demonstrate the procedure for finding the distance between the points $(3\sqrt{2}, -2\sqrt{3})$ and $(\sqrt{2}, 3\sqrt{3})$.

HIGHER ORDER ASSESSMENT TASK

Demonstrate the procedure for finding the distance between the points $(-2a, 3b)$ and $(a, -3b)$.

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PERFORMANCE OBJECTIVE V-2

Demonstrate the procedure for finding the midpoint of a segment, given the endpoints.

1. Demonstrate the procedure for finding the midpoint of the segments whose endpoints are $(5, 3)$ and $(-1, 5)$.
2. Demonstrate the procedure for finding the midpoint of the segments whose endpoints are $(\frac{1}{4}, 3)$ and $(\frac{1}{4}, -7)$.
3. Demonstrate the procedure for finding the midpoint of the segments whose endpoints are $(-2\sqrt{2}, 5)$ and $(-4\sqrt{2}, -1)$.
4. Demonstrate the procedure for finding the midpoint of the segments whose endpoints are $(3\sqrt{2}, -2\sqrt{3})$ and $(\sqrt{2}, 3\sqrt{3})$.

HIGHER ORDER ASSESSMENT TASK:

Find the coordinates of the point A, if point B is $(\frac{3}{2}, 6)$ and the midpoint of $\overline{AB}$ is $(\frac{1}{2}, -2)$.

PERFORMANCE OBJECTIVE V-3

Given three points in a Cartesian plane, determine if they are collinear.

1. Which of the following sets of points are collinear?

a) $\{(0, 3), (1, 7), (-3, -9)\}$

b) $\{(-2, 5), (3, 2), (1, 8)\}$

c) $\{(0, 0), (-3, 3), (5, -5)\}$

2. Which of the following sets of points are collinear?

a) $\{(6, -4), (-1, 3), (-2, 2)\}$

b) $\{(2, 6), (4, -2), (5, 2)\}$

c) $\{(4, 6), (0, 6), (-2, 6)\}$

3. Which of the following sets of points are collinear?

a) $\{(0, 2), (3, 0), (1, 1)\}$

b) $\{(4, 0), (0, 5), (2, \frac{5}{2})\}$

c) $\{(\frac{5}{4}, 1), (-\frac{3}{2}, -2), (1, 4)\}$

4. Which of the following sets of points are collinear?

a) $\{(1, -1), (0, 4), (5, 6)\}$

b) $\{(3, \sqrt{5}), (-1, 5\sqrt{5}), (1, 3\sqrt{5})\}$

c) $\{(5, 3), (5, -2), (5, 1)\}$

PERFORMANCE OBJECTIVE V-4

Determine the perimeter of a triangle, given three non-collinear points.

1. Determine the perimeter of a triangle formed by the points (-2,1), (0,-2), (2, 1).
2. Determine the perimeter of a triangle formed by the points (0, 2), (-2, -2), (1, -5).
3. Determine the perimeter of a triangle formed by the points (4, 0), (1, 3), (-2, 0).
4. Determine the perimeter of a triangle formed by the points $(3, 2\sqrt{3})$, (1, 0), $(4, -\sqrt{3})$.

PERFORMANCE OBJECTIVE V-5

Given three non-collinear points, determine whether the triangle is isosceles and/or a right triangle.

Each of the following sets of points represents the vertices of a triangle. Determine whether the triangle is isosceles and/or whether the triangle is a right triangle.

1. a) (1, 2), (3, 1), (-1, -2)

   b) (-2, 1), (0, -2), (2, 1)

2. a) (2, 3), (6, 3), (6, -1)

   b) $(3, 2\sqrt{3})$, (1, 0), $(4, -\sqrt{3})$

3. a) (1, 4), (10, -8), (-2, -2)

   b) (0, 2), (8, -7), (1, -5)

4. a) (3, 3), (1, 0), (5, 0)

   b) (4, 0), (1, 3), (-2, 0)

PERFORMANCE OBJECTIVE V-6

> Given an equation of a line, state the slope of the lines parallel and perpendicular to that line.

1. a) State the slope of a line parallel to the line $y = 2x + 3$.

   b) State the slope of a line perpendicular to the line $y = 2x + 3$.

2. a) State the slope of a line parallel to the line $2x + 3y = -1$.

   b) State the slope of a line perpendicular to the line $2x + 3y = -1$.

3. a) State the slope of a line parallel to the line $4x + \frac{1}{2}y - 7 = 0$.

   b) State the slope of a line perpendicular to the line $4x + \frac{1}{2}y - 7 = 0$.

4. a) State the slope of a line parallel to the line $y = -3$.

   b) State the slope of a line perpendicular to the line $y = -3$.

PERFORMANCE OBJECTIVE V-7

> Determine an equation of the line parallel to a given line and passing through a given point.

1. Determine the equation of the line parallel to $y = 7$ and passing through $(1, 3)$.
2. Determine the equation of the line parallel to $y = 3x - 1$ and passing through $(1, -2)$.
3. Determine the equation of the line parallel to $2x - 5y = -4$ and passing through $(0, -4)$.
4. Determine the equation of the line parallel to $\frac{x}{2} + \frac{2y}{5} = 1$ and passing through $(0, -2)$.

PERFORMANCE OBJECTIVE V-8

Determine an equation of the line perpendicular to a given line and passing through a given point.

ASSESSMENT TASKS

1. Determine the equation of the line perpendicular to $y = 7$ and passing through $(1, 3)$.

2. Determine the equation of the line perpendicular to $y = 3x - 1$ and passing through $(1, 2)$.

3. Determine the equation of the line perpendicular to $2x - 5y = -4$ and passing through $(-2, 3)$.

4. Determine the equation of the line perpendicular to $\frac{x}{2} + \frac{2y}{5} = -1$ and passing through $(0, -2)$.

PERFORMANCE OBJECTIVE V-9

Translate a direct variation problem into an equation and solve.

1. Write an equation and solve:

   If y varies directly as x and y is 14 when x is 21, what is x when y is 18?

2. Write an equation and solve:

   If y is directly proportioned to x and y is -3 when x is 12, what is y when x is 6?

3. Write an equation and solve:

   If y varies directly as x + 5, and y is 7 when x is 9, what is y when x is 3?

4. Write an equation and solve:

   If y varies directly as 2x - 5 and y is 3 when x is 5, what is x when y is 6?

HIGHER ORDER ASSESSMENT TASK

If y is proportional to x, and ax + by = cx - dy, then y varies directly as x can be restated as ________.

a) $\frac{a - c}{b - d}$

b) $\frac{b - d}{a - c}$

c) $\frac{b + d}{c - a}$

d) $\frac{b + d}{a - c}$

e) $\frac{c - a}{b + d}$

PERFORMANCE OBJECTIVE V-10

Translate a narrative problem involving direct variation into an equation and solve.

1. Write an equation and solve: (Answer may be left in terms of π.).

   If the circumference C of a circle varies directly with the radius r, and C is 4π when r is $\frac{1}{2}$, determine C when r is 8.

2. Write an equation and solve:

   The total cost of 7 gallons of gasoline is $8.61. Determine the cost of 12 gallons of the same gasoline.

3. Write an equation and solve:

   The weight m of an object on the moon varies directly as its weight e on the earth. A person who weighs 15 kilograms on the moon weighs 90 kilograms on the earth. Determine the weight of a person on the moon who weighs 54 kilograms on the earth.

4. Write an equation and solve:

   The dividend on 150 shares of stock is $125.25. What is the dividend on 400 shares of stock?

Translate a narrative problem into an equation and solve.

1. Mitch can prepare and paint a room in 30 hours. Working with his friend Wes, he can finish in 12 hours. If Wes had done the entire job by himself, how long would it have taken?

2. Mary and Ted decide to move themselves across the country. Ted leaves at 9:00 a.m. with the truck, going 45 mph. Mary has a few last minute chores to do, so she doesn't leave in the car until 11:00 a.m. If they decide to stop for the night when Mary, traveling 55 mph, catches up with Ted, how far must they drive before stopping?

3. An automobile radiator holds 12 quarts. How much should be drained and replaced with pure antifreeze to change a 40% antifreeze solution to a 60% solution?

4. The sum of two numbers is 94. If the second is subtracted from three times the first, the result is 58. What are the two numbers?

ANSWERS

V-1

1. $\sqrt{(5-7)^2 + (-1+1)^2} = \sqrt{4} = 2$

2. $\sqrt{(\frac{1}{4} - \frac{1}{4})^2 + (3+7)^2} = \sqrt{100} = 10$

3. $\sqrt{(2\sqrt{2} - \sqrt{2})^2 + (5+1)^2} = \sqrt{2+36} = \sqrt{38}$

4. $\sqrt{(3\sqrt{2} - \sqrt{2})^2 + (-2\sqrt{3} - 3\sqrt{3})^2} = \sqrt{8+75}$

$= \sqrt{83}$

Higher Order Assessment Task

$\sqrt{(-2a - a)^2 + (3b - (-3b))^2}$

$\sqrt{(-3a)^2 + (6b)^2}$

$\sqrt{9a^2 + 36b^2}$

$\sqrt{9(a^2 + 4b^2)}$

$3\sqrt{a^2 + 4b^2}$

V-2

1. $(\frac{5 + -1}{2}, \frac{3 + 5}{2}) = (2, 4)$

2. $(\frac{\frac{1}{4} + \frac{1}{4}}{2}, \frac{3 + -7}{2}) = (\frac{1}{4}, -2)$

3. $(\frac{-2\sqrt{2} + -4\sqrt{2}}{2}, \frac{5 + -1}{2}) = (-3\sqrt{2}, 2)$

4. $(\frac{3\sqrt{2} + \sqrt{2}}{2}, \frac{-2\sqrt{3} + 3\sqrt{3}}{2}) = (2\sqrt{3}, \frac{\sqrt{3}}{2})$

Higher Order Assessment Task

$(-\frac{1}{2}, -10)$

V-3

1. a, c
2. c
3. b
4. b, c

V-4

1. $4 + 2\sqrt{13}$ or $2(2 + \sqrt{13})$
2. $8\sqrt{2} + 2\sqrt{5}$ or $2(4\sqrt{2} + \sqrt{5})$
3. $6 + 6\sqrt{2}$ or $6(1 + \sqrt{2}$
4. $4 + 2\sqrt{3} + 2\sqrt{7}$ or

   $2(2 + \sqrt{3} + \sqrt{7}$

ANSWERS (continued)

V-5

1. a) The triangle is a right triangle.

   b) The triangle is isosceles.

2. a) The triangle is both an isosceles and a right triangle.

   b) The triangle is a right triangle.

3. a) The triangle is a right triangle.

   b) The triangle is neither an isosceles nor a right triangle.

4. a) The triangle is isosceles.

   b) The triangle is both an isosceles and a right triangle.

V-6

1. a) 2

   b) $-\frac{1}{2}$

2. a) $-\frac{2}{3}$

   b) $\frac{3}{2}$

3. a) -8

   b) $\frac{1}{8}$

4. a) 0

   b) no slope or undefined

V-7

1. $y = 3$
2. $3x - y = 5$ or $y = 3x - 5$
3. $2x - 5y = 20$ or $y = \frac{2}{5}x - 4$
4. $5x + 4y = -8$ or $y = -\frac{5}{4}x - 2$

V-8

1. $x = 1$
2. $x + 3y = 7$ or $y \quad \frac{1}{3}x + \frac{7}{3}$
3. $5x + 2y = -4$ or $y = -\frac{5}{2}x - 2$
4. $4x - 5y = 10$ or $y = +\frac{4}{5}x - 2$

ANSWERS (continued)

V-9

1. $y = kx \quad x = 27$
2. $y = kx \quad y = -\frac{3}{2}$
3. $y = k(x + 5) \quad y = 4$
4. $y = k(2x - 5) \quad y = \frac{15}{2}$

Higher Order Assessment Task:

e)

V-10

1. $C = kr$ or $\frac{C}{r} = k$

   64π

2. $C = Gk$ where C = cost of gas
   and
   G = number of gallons of gas

   $14.76

3. $m = k \cdot e$

   9 kilograms

4. $D = S \cdot k$ where

   D = dividend and

   S = shares

   $334.00

V-11

1. 20 hours
2. 495 miles
3. 4 quarts
4. 38 and 56

UNIT VI - SOLVING SECOND DEGREE EQUATIONS IN ONE VARIABLE

PURPOSE

Since first degree equations have been studied extensively, this unit extends these concepts by solving second degree equations.

OVERVIEW

The student begins by solving the square of a binomial, then examines various methods for determining the roots of quadratic open sentences, concluding with fractional equations with extraneous roots.

SUGGESTIONS TO THE TEACHER

The process of checking solutions should be emphasized before the study of fractional equations with extraneous roots.

Answers to the assessment measures of objective six are given in correct set notation: the use of $\cup$, $\cap$ for "or," "and."

Objective seven may be used, even if the sum and product of the roots to determine the equation is not emphasized as the method.

Computer Applications: BASIC BASIC, Coan, pp. 114-115 (Section 8-1); Algebra 2 and Trigonometry, Dolciani (1980), pp. 285, 308; Algebra Two with Trigonometry, Foster, pp. 209, 250, 253, 536; Computer Programming in the BASIC Language, Golden, pp. 66, 67, 95.

The time allocation for the unit is 15 days.

VOCABULARY

coefficient
constant term
discriminant
extraneous root(s)
linear term
quadratic term
root(s) of an equation

UNIT VI - SOLVING SECOND DEGREE EQUATIONS IN ONE VARIABLE

PERFORMANCE OBJECTIVES

1. Determine the roots of a quadratic equation involving the square of a binomial.
2. Determine the roots of a quadratic equation by factoring.
3. Determine the roots of a quadratic equation by completing the square.
4. Determine the roots of a second degree equation using the quadratic formula.
5. Determine the nature of the roots of a quadratic equation by examining the discriminant.
6. Use quadratic and fractional open sentences to solve narrative problems.
7. Given a quadratic inequality, determine its solution set over R and graph it.
8. Given the solutions of a quadratic equation with integral coefficients, use the sum and product of the roots to determine the equation.
9. Solve fractional equations, checking for extraneous roots.

UNIT VI - SOLVING SECOND DEGREE EQUATIONS IN ONE VARIABLE

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	251-255	284	--	--	--	234-236	194-195	309-312	282-283
2	140-143	283	186-188	166-167	370-372	69-70	186-190	185-187	281-283
3	237-240	284-285	286-288	168-170	195 403	237-240	194-195	309-313 335-338	273-274 288
4	274-276	284-287	285-288	172-174 184-185	374-378	241-244	196-201	313-316	285-288
5	281-285	300-302	305-308	175-177	379-380	245-247	221-224	322-324	289-291
6	268 277	286 208-211	188-189 206-208	182-184	372-373 378;386 284-288	243-244	199-201	317	292-295
7	285-287	313-314	319-321	206-209	--	--	205-207	189-191	300-301
8	279-281	304-305	309-310	178-180	381-383	248-249	202-204	318-321	--
9	184-187	208-211	205-206	290-292	278-283	244	--	236-239	139-141

PERFORMANCE OBJECTIVE VI-1

Determine the roots of a quadratic equation involving the square of a binomial.

1. Solve the equation $(x - 4)^2 = 9$
2. Solve the equation $(x - 7)^2 = 15$
3. Solve the equation $(x + 3)^2 - 2 = 0$
4. Solve the equation $(x + 5)^2 + 25 = 0$

PERFORMANCE OBJECTIVE VI-2

Determine the roots of a quadratic equation by factoring.

1. By factoring, determine the solution set for x in the equation $2x^2 - 5 = 4$.
2. By factoring, determine the solution set for x in the equation $x^2 + 2x - 15 = 0$.
3. By factoring, determine the solution set for x in the equation $3x^2 + 19x - 14 = 0$.
4. By factoring, determine the solution set for x in the equation $x(3x - 7) = x - 5$.
5. By factoring, determine the solution set for x in the equation $x^2 + 9 = 0$.

PERFORMANCE OBJECTIVE VI-3

Determine the roots of a quadratic equation by completing the square.

1. Determine x by completing the square $x^2 + 4x - 5 = 0$.
2. Determine x by completing the square $2x^2 = 12x - 5$.
3. Determine x by completing the square $x^2 + \sqrt{3}x + 1 = 0$.
4. Determine x by completing the square $9x^2 - 6x = 17$.
5. Determine x by completing the square $x^2 - 6x + 13 = 0$.

PERFORMANCE OBJECTIVE VI-4

Determine the roots of a second degree equation using the quadratic formula.

1. Use the quadratic formula to solve $16x^2 - 2x - 3 = 0$.
2. Use the quadratic formula to solve $6x^2 + x\sqrt{3} - 3 = 0$.
3. Use the quadratic formula to solve $3x^2 - 8x + 2 = 0$.
4. Use the quadratic formula to solve $2x - 2x^2 - 5 = 0$.

HIGHER ORDER ASSESSMENT TASK

Use the quadratic formula to solve $x^4 - 5x^2 + 4 = 0$.

PERFORMANCE OBJECTIVE VI-5

Determine the nature of the roots of a quadratic equation by examining the discriminant.

1. By examining the discriminant, determine which one of the following <u>best</u> describes the roots of the equation $3x^2 + 8x + 2 = 0$.

   a) one double real root

   b) two complex conjugate roots

   c) two rational roots

   d) two real roots

   e) one double rational root

2. By examining the discriminant, determine which one of the following <u>best</u> describes the roots of the equation $x^2 + 2\sqrt{2}x - 2 = 0$.

   a) one double real root

   b) two complex conjugate roots

   c) two rational roots

   d) two real roots

   e) one double rational root

3. By examining the discriminant, determine which one of the following <u>best</u> describes the roots of the equation $2x^2 - 4x + 7 = 0$

   a) one double real root

   b) two complex conjugate roots

   c) two rational roots

   d) two real roots

   e) one double rational root

PERFORMANCE OBJECTIVE VI-5

Determine the nature of the roots of a quadratic equation by examining the discriminant.

4. By examining the discriminant, determine which one of the following best describes the roots of the equation $x^2 - 6x + 9 = 0$.

   a) one double real root

   b) two complex conjugate roots

   c) two rational roots

   d) two real roots

   e) one double rational root

5. By examining the discriminant, determine which one of the following best describes the roots of the equation $8x^2 + 8x + 2 = 0$.

   a) one double real root

   b) two complex conjugate roots

   c) two rational roots

   d) two real roots

   e) one double rational root

PERFORMANCE OBJECTIVE VI-6

Use quadratic and fractional open sentences to solve narrative problems.

1. The area of a rectangle is 108 square centimeters. If the width is 3 centimeters shorter than the length, determine the dimensions of the rectangle.
2. The sum of the squares of two consecutive integers is 265. Determine the two integers.
3. One side of a right triangle is 1 centimeter less than the hypotenuse and 7 centimeters more than the other side. Determine the lengths of each side.
4. Two positive integers differ by 9. Their product is 112. Determine the two integers.
5. A rectangular parking lot with dimensions 20 meters by 30 meters is enlarged by extending each side of the lot an equal amount. If the area of the lot is doubled, determine the new dimensions.
6. The base of a triangle is 4 feet less than the altitude, and the area of the triangle is at least 48 square feet. Determine the length of the base.

PERFORMANCE OBJECTIVE VI-7

> Given a quadratic inequality, determine its solution set over R and graph it.

1. Determine the solution set for x in the inequality $(x + 1)^2 > 9$ and graph the solution.
2. Determine the solution set for x in the inequality $3x^2 + 8x - 3 > 0$ and graph the solution.
3. Determine the solution set for x in the inequality $x^2 + 2x + 1 < 0$ and graph the solution.
4. Determine the solution set for x in the inequality $2x^2 - 11x + 12 \leq -3$ and graph the solution.

PERFORMANCE OBJECTIVE VI-8

> Given the solutions of a quadratic equation with integral coefficients, use the sum and product of the roots to determine an equation.

1. Use the sum and product of the roots $\{4, -7\}$ to write an equation with integral coefficients.
2. Use the sum and product of the roots $\{\frac{1}{2}, \frac{1}{3}\}$ to write an equation with integral coefficients.
3. Use the sum and produce of the roots $\{2 + \sqrt{3}, 2 - \sqrt{3}\}$ to write an equation with integral coefficients.
4. Use the sum and product of the roots $\{-\frac{1}{2} + \frac{\sqrt{3}}{2}, -\frac{1}{2} - \frac{\sqrt{3}}{2}\}$ to write an equation with integral coefficients.

HIGHER ORDER ASSESSMENT TASK:

If one root of the equation $x^2 + bx - 14 = 0$ is 7, find the other root and the value of b.

PERFORMANCE OBJECTIVE VI-9

Solve fractional equations; checking for extraneous values.

1. Solve and check, eliminating any extraneous roots from the solution set of the equation $\frac{x + 3}{x + 2} - \frac{x + 1}{x + 3} = \frac{x^2 - 3}{x^2 + 5x + 6}$.

2. Solve and check, eliminating any extraneous roots from the solution set $\frac{1}{y + 2} + \frac{1}{y - 2} = \frac{y^2 + 1}{y^2 - 4}$.

3. Solve and check, eliminating any extraneous roots from the solution set of the equation $\frac{x + 7}{x + 4} = \frac{x - 2}{4}$.

4. Solve and check, eliminating any extraneous roots from the solution set of $\frac{4}{x + 4} - \frac{4}{x - 4} = 1$.

UNIT VI - SOLVING SECOND DEGREE EQUATIONS IN ONE VARIABLE

ANSWERS

VI-1

1. $\{7, 1\}$
2. $\{7 - \sqrt{15}, 7 + \sqrt{15}\}$
3. $\{-3 + \sqrt{2}, -3 - \sqrt{2}\}$
4. $\{-5 + 5i, -5 - 5i\}$

VI-2

1. $\{\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\}$
2. $\{-5, 3\}$
3. $\{\frac{2}{3}, -7\}$
4. $\{\frac{5}{3}, 1\}$
5. $\{3i, -3i\}$

VI-3

1. $\{1, -5\}$
2. $\{3 + \frac{\sqrt{26}}{2}, 3 - \frac{\sqrt{26}}{2}\}$ or

   $\{\frac{6 + \sqrt{26}}{2}, \frac{6 - \sqrt{26}}{2}\}$
3. $\{\frac{-\sqrt{3}}{2} + \frac{i}{2}, \frac{-\sqrt{3}}{2} - \frac{i}{2}\}$
4. $\{\frac{1}{3} + \sqrt{2}, \frac{1}{3} - \sqrt{2}\}$

   $\{\frac{1 + 3\sqrt{2}}{3}, \frac{1 - 3\sqrt{2}}{3}\}$
5. $\{3 + 2i, 3 - 2i\}$

VI-4

1. $\{\frac{1}{2}, -\frac{3}{8}\}$
2. $\{-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}$
3. $\{\frac{4 + \sqrt{10}}{3}, \frac{4 - \sqrt{10}}{3}\}$
4. $\{\frac{1}{2} + \frac{3i}{2}, \frac{1}{2} - \frac{3i}{2}\}$

Higher Order Assessment Task:

$\pm 2, \pm 1$

VI-5

1. d
2. d
3. b
4. e

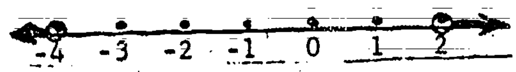
VI-6

1. 12 cm by 9 cm
2. 11, 12; -11, -12
3. 5 cm, 12 cm, 13 cm
4. 16, 7
5. 30 m by 40 m
6. base $\geq$ 8 feet

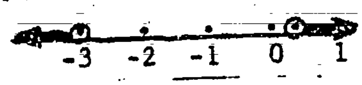
ANSWERS (continued)

VI-7

1. $\{x \mid x > 2 \cup x < -4\}$

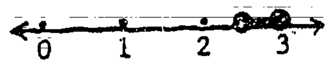


2. $\{x \mid x > \frac{1}{3} \cup x < -3\}$



3. $\emptyset$

4. $\{x \mid \frac{5}{2} \leq x \leq 3\}$



VI-8

1. $x^2 + 3x - 28 = 0$
2. $6x^2 - 5x + 1 = 0$
3. $x^2 - 4x + 1 = 0$
4. $2x^2 + 2x - 1 = 0$

Higher Order Assessment Task:

root = -2

b = -5

VI-9

1. x = 5; x = -2

   {5}

2. {1}

3. $\{1 + \sqrt{37}, 1 - \sqrt{37}\}$

4. x = -4i; x = 4i

   {4i, -4i}

UNIT VII - SOLVING EQUATIONS OF HIGHER DEGREE

PURPOSE

Polynomials of higher degree are studied in order that equations not readily factorable can be solved.

OVERVIEW

The procedures for factoring and solving quadratic equations are extended. Analytical methods are used to solve higher degree equations over the complex number system.

SUGGESTIONS TO THE TEACHER

In the objectives, the term "synthetic division" is used if the usual process requires factoring; the term "synthetic substitution" is used if the process involves finding roots of an equation and values of a function.

It is important that the student understand the Remainder Th.orem, the Factor Theorem, and the relationship between the degree of the polynomial equation and the number of complex roots of the equation.

The enrichment for the unit includes Descartes' Rule of Signs, upper and lower bounds of real roots, graphing of functions of higher degree, and estimating real zeros of a function.

Computer Applications: Algebra 2 and Trigonometry, Dolciani (1978), pp. 317, 322, 326; Algebra 2 and Trigonometry, Dolciani (1980), pp. 325, 331, 334; Algebra Two with Trigonometry, Foster, p. 171; Computer Programming and the BASIC Language, Golden, pp. 30, 66, 171-179; Algebra Two and Trigonometry, Keedy, p. 475; Using Advanced Algebra, Travers, p. 498.

An entire chapter of BASIC BASIC, by James S. Coan, is devoted to solving polynomial equations; developing programs for determining integral roots, real roots, and finally complex roots (pp. 148-167).

The time allocation for this unit is approximately ten days.

VOCABULARY

synthetic division
synthetic substitution
zeros of a function
depressed polynomial (reduced polynomial)

UNIT VII - SOLVING EQUATIONS OF HIGHER DEGREE

PERFORMANCE OBJECTIVES

1. Use synthetic division to determine the quotient and remainder when a polynomial is divided by x - a.

2. Use synthetic substitution to determine the value of a given polynomial function.

3. Use synthetic substitution to determine which given numbers are roots of a polynomial equation.

4. Use synthetic division to determine which given binomials are factors of a polynomial expression.

5. List all possible rational roots of a given integral polynomial equation by determine p/q, where p is an integral factor of the constant term and q is an integral factor of the leading coefficient. (Rational Root Theorem)

6. Use the Rational Root Theorem and synthetic substitution to determine all rational roots of an equation with integral coefficients.

7. Use synthetic substitution and depressed polynomials to determine all roots of a polynomial equation.

8. Given at least one complex root of a polynomial equation, identify the other complex root.

ENRICHMENT

1. Use Descartes' Rule of Signs to determine the number of real roots possible in a given polynomial equation.

2. Use Descartes' Rule of Signs to determine the nature of the roots of a given polynomial equation.

3. Determine the upper and lower bounds for the real roots of a given polynomial equation.

4. Use synthetic substitution to graph a polynomial function over the set of real numbers and estimate any real zeros.

VOCABULARY

synthetic division
synthetic substitution
zeros of a function
depressed polynomial (reduced polynomial)

UNIT VII - SOLVING EQUATIONS OF HIGHER DEGREE

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	527-529	318-321	327-330	125-128	462-463	71	339-342	195-197	480-482
2	527-529	315-317	322-324	251-253	464	64	344 346	195-197 255-257	483-485
3	529-532	319-321	--	260-262	464-465	--	346	198-199	--
4	529-532	319-321	--	257-259	466-467	65 71	339-342 344-346	195-199	483-486
5	255-258	265-267	261-264	254-256	471-474	--	--	275-276	487-492
6	255-259	320-321	261-264	254-256	471-474	--	--	275-276	487-492
7	532-533	320-321	--	257-259	472	71	346	198-199	494-497
8	533-535	323-324	--	257-259	467-469	--	--	300-313	494-497

UNIT VII - SOLVING EQUATIONS OF HIGHER DEGREE

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1979)	Travers (1978)
ENRICHMENT									
1	535-537	--	--	258	--	--	--	347-348	--
2	535-537	--	--	260-262	--	--	--	347-349	--
3	541-542	--	--	259	--	--	--	348-349	--
4	537-541	325-326	--	264-266	--	--		422-425	--

PERFORMANCE OBJECTIVE VII-1

Use synthetic division to determine the quotient and remainder when a polynomial is divided by $x - a$.

1. Use synthetic division to determine the quotient and the remainder, when $3x^4 - 13x^3 + 9x^2 + 11x - 6$ is divided by $x - 3$.

2. Use synthetic division to determine the quotient and the remainder, when $x^4 - 10x^2 + 5x - 1$ is divided by $x + 3$.

3. Use synthetic division to determine the quotient and the remainder, when $3x^4 + 2x^3 - x^2 + 5x - 1$ is divided by $x - \sqrt{2}$.

4. Use synthetic division to determine the quotient and the remainder, when $x^4 + 2x^3 - x^2 + 5x - 1$ is divided by $x + 2i$.

5. Use synthetic division to determine the quotient and the remainder, when $2x^4 + 2x^3 - 15x^2 - 5x - 1$ is divided by $x - (3 + i)$.

PERFORMANCE OBJECTIVE VII-2

Use synthetic substitution to determine the value of a given polynomial function.

1. If $P(x) = 2x^4 - 5x^3 + 7x^2 - 10x + 3$, determine $P(-3)$ and $P(5)$ by using synthetic substitution.

2. If $P(x) = x^3 + 2x^2 + 6$, determine $P(-1)$ and $P(-10)$ by using synthetic substitution.

3. If $P(x) = x^4 + 3x^2 + 5$, determine $P(2)$ and $P(-2)$ by using synthetic substitution.

4. If $P(x) = 3x^3 - 16x^2 + 17x - 4$, determine $P(\frac{1}{3})$ and $P(4)$ by using synthetic substitution.

PERFORMANCE OBJECTIVE VII-3

> Use synthetic substitution to determine which given numbers are roots of a polynomial equation.

1. Use synthetic substitution to determine which of the numbers 3, -1, $\frac{1}{2}$, 4 are roots of the equation $2x^3 - 3x^2 - 11x + 6 = 0$.

2. Use synthetic substitution to determine which of the numbers -2, -1, 1, 2 are roots of the equation $x^3 + 3x^2 - 2x - 6 = 0$.

3. Use synthetic substitution to determine which of the numbers -3, -2, -1, 1, 2 are roots of the equation $3x^3 - x^2 - 20x - 12 = 0$.

4. Use synthetic substitution to determine which of the numbers $\sqrt{3}$, $\sqrt{2}$, $-\sqrt{2}$, $-\sqrt{3}$ are roots of the equation $x^3 + 2x^2 - 3x - 6 = 0$.

PERFORMANCE OBJECTIVE VII-4

> Use synthetic division to determine which given binomials are factors of a polynomial expression.

1. Use synthetic division to determine which of the binomials $(x + 2)$, $(x - 1)$, $(x - 2)$ and $(x - 5)$ are factors of the polynomial $x^4 - 2x^2 - 7x + 6$.

2. Use synthetic division to determine which of the binomials $(x + 3)$, $(x - 2)$, $(x + 1)$, and $(x - 1)$ are factors of the polynomial $x^3 - 3x^2 + x - 3$.

3. Use synthetic division to determine which of the binomials $(x - 3)$, $(x - 2)$, $(x + 1)$, and $(x + 3)$ are factors of the polynomial $x^4 - x^3 - 11x^2 + 9x + 18$.

4. Use synthetic division to determine which of the binomials $(x - \sqrt{2})$, $(x + \sqrt{2})$, $(x - \sqrt{5})$, and $(x + \sqrt{5})$ are factors of the polynomial $x^4 + 4x^3 + 3x^2 - 8x - 10$.

PERFORMANCE OBJECTIVE VII-5

List all possible rational roots of a given integral polynomial equation by determining $\frac{p}{q}$, where p is an integral factor of the constant term and q is an integral factor of the leading coefficient (Rational Root Theorem).

1. List all possible rational roots of the equation $x^3 - 2x^2 + 4x + 12 = 0$.
2. List all possible rational roots of the equation $2x^4 + x^3 + 9x^2 - 8x - 24 = 0$.
3. List all possible rational roots of the equation $4x^3 - 2x^2 - x + 16 = 0$.

PERFORMANCE OBJECTIVE VII-6

Use the Rational Root Theorem and synthetic substitution to determine all rational roots of an equation with integral coefficients.

1. Use the Rational Root Theorem and synthetic division to determine all rational roots of the equation $4x^3 + 28x^2 - 31x + 8 = 0$.
2. Use the Rational Root Theorem and synthetic division to determine all rational roots of the equation $x^4 + 9x^3 + 11x^2 - 21x = 0$.
3. Use the Rational Root Theorem and synthetic division to determine all rational roots of the equation $6x^3 - 2x^2 - 76x - 48 - 0$.
4. Use the Rational Root Theorem and synthetic division to determine all rational roots of the equation $2x^3 - x^2 - 6x + 3 = 0$.

PERFORMANCE OBJECTIVE VII-8

Given at least one complex root of a polynomial equation with real coefficients, identify the other complex zero.

1. If $1 + i$ is a root of the equation $2x^3 - 3x^2 + 2x + 2 = 0$, what other number is also a root?

   a) $-1 + i$

   b) $-1 - i$

   c) $1 - i$

2. If $1 + 3i$ is one root of the equation $x^4 - 3x^3 + 6x^2 + 2x - 60 = 0$, state one other complex root.

3. If $3i$ and $1 - 2i$ are roots of an equation of degree four, what are the other roots?

4. If $3 + i$ is a root of the equation $x^4 - 6x^3 + 14x^2 - 24x + 40 = 0$, what other number is also a root?

   a) $3 - i$

   b) $-3 + i$

   c) $-3 - i$

PERFORMANCE OBJECTIVE VII-7

Use synthetic substitution and depressed polynomials to determine all roots of a polynomial equation.

1. Use synthetic substitution and depressed polynomials to determine all roots of the polynomial $x^3 - 7x - 6 = 0$.
2. Use synthetic substitution and depressed polynomials to determine all roots of the polynomial $x^4 + 2x^3 - 11x^2 - 6x + 24 = 0$.
3. Use synthetic substitution and depressed polynomials to determine all roots of the polynomial $4x^3 + 28x^2 - 31x + 8 = 0$.
4. Use synthetic substitution and depressed polynomials to determine all roots of the polynomial $2x^4 - 17x^3 + 58x^2 - 77x + 26 = 0$.

ENRICHMENT 1

Use Descartes' Rule of Signs to determine the number of real roots possible in a given polynomial equation.

1. Use Descartes' Rule of Signs to determine the number of positive real roots possible in the equation $x^4 + 3x^3 + 2x^2 + 5x + 7 = 0$.

   a) zero d) three

   b) one e) four

   c) two f) five

2. Use Descartes' Rule of Signs to determine the number of negative real roots possible in the equation $x^4 + 3x^3 + 2x^2 + 5x + 7 = 0$.

   a) zero d) three

   b) one e) four

   c) two f) five

3. Use Descartes' Rule of Signs to determine the number of positive real roots possible in the equation $x^5 + 5x^4 - 3x^3 - 5x^2 + 2x - 7 = 0$.

   a) zero d) three

   b) one e) four

   c) two f) five

4. Use Descartes' Rule of Signs to determine the number of negative real roots possible in the equation $x^5 + x^3 - x^2 - x + 7 = 0$.

   a) zero d) three

   b) one e) four

   c) two f) five

ENRICHMENT 2

> Use Descartes' Rule of Signs to determine the nature of the roots of a given polynomial equation.

1. Use Descartes' Rule of Signs to determine the nature of the roots of $x^4 + 2x^3 - 3x^2 - x + 8 = 0$.

Positive Real	Negative Real	Imaginary

2. Use Descartes' Rule of Signs to determine the nature of the roots of $2x^4 - 3x^3 + x^2 - 4x + 6 = 0$.

Positive Real	Negative Real	Imaginary

3. Use Descartes' Rule of Signs to determine the nature of the roots of $3x^5 - 2x^3 + x^2 - 9 = 0$.

Positive Real	Negative Real	Imaginary

ENRICHMENT 3

Determine the upper and lower bounds for the real roots of a given polynomial equation.

1. Determine the upper and lower bounds for the real roots of

   $x^4 + 2x^3 - 3x^2 - x + 8 = 0.$

2. Determine the upper and lower bounds for the real roots of

   $2x^4 - 3x^3 + x^2 - 4x + 6 = 0.$

3. Determine the upper and lower bounds for the real roots of

   $3x^5 - 2x^3 + x^2 - 9 = 0.$

ENRICHMENT 4

Use synthetic substitution to graph a polynomial function over the set of real numbers and estimate any real zeros.

1. Use synthetic substitution to graph P(x) and estimate the real zeros.

   $$P(x) = x^3 + 2x^2 - 5x - 6$$

2. Use synthetic substitution to graph P(x) and estimate the real zeros.

   $$P(x) = x^4 - 3x^3 + 2x^2 - x + 3$$

3. Use synthetic substitution to graph P(x) and estimate the real zero between 0 and 1.

   $$P(x) = x^3 - 5x^2 + 6x - 1$$

ANSWERS

VII-1

1. Quotient: $(3x^3 - 4x^2 - 3x + 2)$; remainder 0
2. Quotient: $(x^3 - 5x^2 + 15x - 70)$; remainder 349
3. Quotient $[3x^3 + (3\sqrt{2} + 2)x^2 + (5 + 2\sqrt{2})x + (9 + 5\sqrt{2})]$; remainder $9\sqrt{2} + 9$
4. $x^3 + (2 - 2i)x^2 + (-5 - 4i)x + (-3 + 10i)$; remainder $(19 + 6i)$
5. Quotient: $2x^3 + (8 + 2i)x^2 + (7 + 14i)x + (2 + 49i)$; remainder $(-44 + 149i)$

VII-2

1. $P(-3) = 393$

   $P(5) = 753$
2. $P(-1) = 7$

   $P(-10) = -794$
3. $P(2) = 33$

   $P(-2) = 33$
4. $P(\frac{1}{3}) = 0$

   $P(4) = 0$

VII-3

1. Solution set: $\{3, \frac{1}{2}\}$
2. Solution set: ϕ
3. Solution set: $\{-2\}$
4. Solution set: $\{\sqrt{3}, -\sqrt{3}\}$

VII-4

1. The only factor is <u>$(x - 2)$</u>
2. <u>None</u> are factors
3. <u>All</u> are factors
4. The factors are $(x - \sqrt{2})$ and $(x + \sqrt{2})$

VII-5

1. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
2. $\pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$
3. $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2, \pm 4, \pm 8, \pm 16$
4. $\pm 1, \pm\frac{1}{5}, \pm 5, \pm 7, \pm\frac{7}{5}, \pm 35$

VII-6

1. Solution set: $\{\frac{1}{2}, -8\}$
2. Solution set: $\{0, 1, -3, -7\}$
3. Solution set: $\{-3, -\frac{2}{3}, 4\}$
4. Solution set: $\{\frac{1}{2}\}$

ANSWERS (continued)

VII-7

1. $\{-1, -2, 3\}$
2. $\{2, -4, \sqrt{3}, -\sqrt{3}\}$
3. $\{\frac{1}{2}, -8\}$
4. $\{\frac{1}{2}, 2, 3 + 2i, 3 - 2i\}$

VII-8

1. c
2. $1 - 3i$
3. $-3i, 1 + 2i$
4. a

Enrichment 1

1. a
2. a , c , e
3. b , d
4. b

Enrichment 2

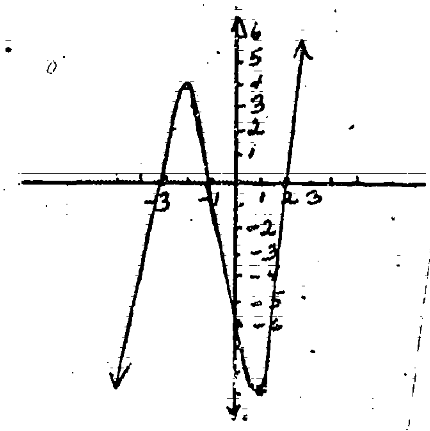
	Positive Real	Negative Real	Imaginary
1.	2	2	0
	2	0	2
	0	2	2
	0	0	4
2.	4	0	0
	2	0	2
	0	0	4
3.	3	2	0
	3	0	2
	1	2	2
	1	0	4

Enrichment 3

	Upper Bound	Lower Bound
1.	2	-3
2.	2	1
3.	2	-1

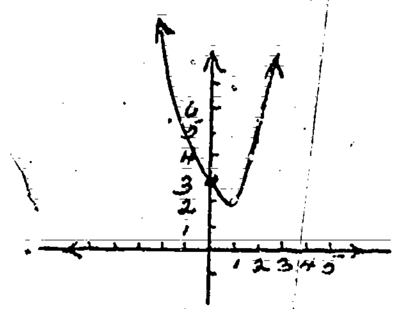
Enrichment 4

1.



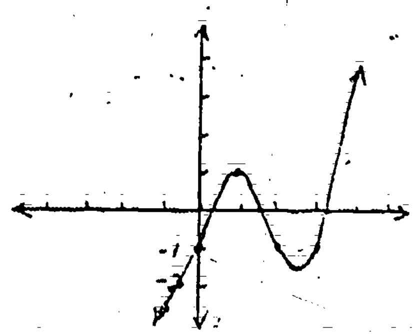
The zeros of $P(x)$ are $-3, -1, 2$.

2.



There are no real roots.

3.



There is a root between 0 and 1 (approximately 0.2).

UNIT VIII - CONIC SECTIONS

PURPOSE

Conic sections unify the study of lines and quadratic equations in one variable with the right circular cone.

OVERVIEW

The student is expected to graph and analyze second degree open sentences in more than one variable.

SUGGESTIONS TO THE TEACHER

Conic sections may be introduced by using examples from the environment.

Formal definitions for the conic sections are considered important, even though they are not included as performance objectives.

It is recommended that advanced students demonstrate the procedure for deriving the equations of each of the conic sections. The Enrichment introduces eccentricity in determining the equations of conics.

Computer Applications: BASIC BASIC, Coan, pp. 114-123; Algebra Two with Trigonometry, Foster, pp. 198, 216; Computer Programming in the BASIC Language, Golden, pp. 66-67, 95-97, 170; Algebra Two with Trigonometry, Payne, pp. 520-521.

The time allocation for this unit is approximately 20 days.

VOCABULARY

conic section	inverse variation
circle	combined variation
ellipse	joint variation
parabola	concavity
hyperbola	asymptotes
directrix	minimum point
focus (foci)	maximum point
axis of symmetry	axis (axes)

PERFORMANCE OBJECTIVES

1. Determine the equation of a parabola, given the focus and directrix.
2. Given a quadratic relation defined by $x = ay^2 + by + c$ or $f(x) = ax^2 + bx + c$:
   a) Write it in $x = a(y - k)^2 + h$ or $f(x) = a(x - h)^2 + k$ form.
   b) Determine the axis of symmetry.

c) Determine the coordinates of the vertex.

d) If the relation is a function, state the maximum or minimum value.

e) Sketch the graph of the relation.

3. Given a narrative problem involving the maximum or minimum value of a function, translate to an equation and solve.

4. Given its center and radius, determine the equation of a circle.

5. Sketch the graph of a relation in the form $(x - h)^2 + (y - k)^2 = r^2$.

6. Given the coordinates of the foci and the sum of the lengths of the focal radii, determine the equation of the ellipse.

7. Given an equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$, sketch the graph of the relation.

8. Given the coordinates of the foci and the absolute value of the difference of the focal radii, determine the equation of the hyperbola.

9. Given an equation in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, sketch the graph of the hyperbola.

10. Given a quadratic equation of the form $Ax^2 + By^2 + Cz + Dy + E = 0$:

a) Identify the conic.

b) Write the equation in standard form.

c) State the appropriate basic properties (for the circle, its radius and center; for the parabola, its vertex and axis of symmetry; for the ellipse, its x- and y-intercepts and the lengths of its major and minor axes; for the hyperbola, its intercepts and asymptotes).

d) Sketch the graph.

11. Graph the subsets of the plane defined by a quadratic inequality.

12. Sketch a hyperbola in the form $xy = k$, where $k \neq 0$.

13. Given a narrative problem involving inverse variation, translate to an equation and solve.

14. Given a narrative problem involving joint or combined variations, translate into an equation and solve.

ENRICHMENT

Determine the equation for a conic, given the focus, directrix, and eccentricity.

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Colciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	309-312	341-343	350-353	217-219	441 444	367-368	171-183 279-282	330-338 365	313-315
2	226-230 237-240 309-311	307-312	312-318	192-194	403-408	215-230 336-368	281-282	330-333 335-342 362-365	269-278 313-315
3	227-228	312-313	318-319	203-205	404-406	215 217 228-230	176 179	339-343	271 276-278
4	06-308	339-340	348-350	220-222	429-431	358-361	288-289 291	359-361	310-312
5	306-308	340	348-350	220-222	430-431	360	289 291	359-361	310-312
6	312-315	344-347	353-356	224-227	432-435	362-366	292-296	366-369	316-319
7	312-315	346	353-356	224-227	432-435	363-365	293-298	366-369	[illegible]-319
8	315-319	347-351	358-362	228-231	436-439	369-371	300-304	369-372	320-323
9	315-318	349-350	358-362	228-231	436-439	370-371	301-304	369-372	320-323
10	306-318 Higher Order 315	340; 343 346; 351	--	232-234	431;435 440;444	365;366 367;37[illegible] 373	279-304 HO: 302 297-300	359-372	319;323 324-327

VIII-3

UNIT VIII - CONIC SECTIONS

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
11	242-243 306-308 312-315	341; 343 347; 352	319-321	206-209	--	383	208-210	359-361 362-365 366-369	299-301
12	319-321	353	364	--	439-440	372-373	--	266-269 272	330-332
13	319-323	353-356	363-367	297-300	415-417	373-377	--	266-270	332-333
14	319-323	354-356	365-367	297-300	415-417	375-377	--	266-270	333
ENRICHMENT	337	--	--	--	--	--	316-317	386	--

PERFORMANCE OBJECTIVE VIII-1

Determine the equation of a parabola, given the focus and directrix.

1. Determine the equation of the parabola with focus (0, 2) and directrix $y = -10$.

2. Determine the equation of the parabola with focus (-2, -1) and directrix $x = 2$.

3. Determine the equation of the parabola with focus (4, 0) and directrix $y = -4$.

4. Determine the equation of the parabola with focus (6, -2) and directrix $y = 6$.

Given a quadratic relation defined by $x = ay^2 + by + c$ or $f(x) = ax^2 + bx + c$:

a) Write it in $x = a(y - k)^2 + h$ or $f(x) = a(x - h)^2 + k$ form.

b) Determine the axis of symmetry.

c) Determine the coordinates of the vertex.

d) If the relation is a function, state the maximum or minimum value.

e) Sketch the graph of the relation.

1. Given the quadratic relation defined by: $x = 3y^2 + 30y + 68$:

   a) Write it in $x = a(y - k)^2 + h$ form.

   b) Determine the axis of symmetry.

   c) Determine the coordinates of the vertex.

   d) If the relation is a function, state the maximum or minimum value.

   e) Sketch the graph of the relation.

2. Given the quadratic relation defined by $f(x) = 3x^2 - 6x + 8$:

   a) Write it in $f(x) = a(x - h)^2 + k$ form.

   b) Determine the axis of symmetry.

   c) Determine the coordinates of the vertex.

   d) If the relation is a function, state the maximum or minimum value.

   e) Sketch the graph of the relation.

3. Given the quadratic relation defined by $x = -2y^2 + 6y - 7$:

   a) Write it in $x = a(y - k)^2 + h$ form.

   b) Determine the axis of symmetry.

   c) Determine the coordinates of the vertex.

   d) If the relation is a function, state the maximum or minimum value.

   e) Sketch the graph of the relation.

PERFORMANCE OBJECTIVE VIII-2 (Continued)

4. Given the quadratic relation defined by $f(x) = -2x^2 - 12x - 22$:

   a) Write it in $f(x) = a(x - h)^2 + k$ form.

   b) Determine the axis of symmetry.

   c) Determine the coordinates of the vertex.

   d) If the relation is a function, state the maximum or minimum value.

   e) Sketch the graph of the relation.

PERFORMANCE OBJECTIVE VIII-3

Given a narrative problem involving the maximum or minimum value of a function, translate to an equation and solve.

1. W. & G. Realty estimates that the month p" in dollars from a building "x" stories high can be found by $= -2x^2 + 44x$. What height building would give the company the most profit?

2. From all pairs of numbers whose difference is 40, determine the two whose product is the least.

3. The Smith family used 200 feet of fencing to enclose their rectangular yard. What are the dimensions and the area of the largest pool with a deck that can be built and enclosed by this fencing?

4. Dr. Helen West and her associates see 100 patients a day, and each patient pays $20.00 for the office visit. She estimates that the group will lose four patients for each $1.00 increase in the office fee. Find the most profitable fee for them to charge.

PERFORMANCE OBJECTIVE VIII-4

Given its center and radius, determine the equation of a circle.

1. A circle has center at (0, 0) and a radius of 6; write its equation.
2. A circle has its center at the origin and its radius equals 7; write its equation.
3. A circle has its center at (-5, 7) and its radius equals 5; write its equation.
4. A circle has its center at (-1, -4) and its radius equals $\sqrt{3}$; write its equation.

PERFORMANCE OBJECTIVE VIII-5

Sketch the graph of a relation in the form $(x - h)^2 + (y - k)^2 = r^2$.

1. Sketch the graph of $x^2 + y^2 = 25$.
2. Sketch the graph of $(x - 2)^2 + y^2 = 16$.
3. Sketch the graph of $(x + 3)^2 + (y - 2)^2 = 9$.
4. Sketch the graph of $(x - 4)^2 + (y + 3)^2 = 4$.

PERFORMANCE OBJECTIVE VIII-6

Given the coordinates of the foci and the sum of the lengths of the focal radii, determine the equation of the ellipse.

1. Determine the equation of the ellipse which has (3,0) and (-3,0) as coordinates of its foci and 12 as the sum of the focal radii.

2. Determine the equation of the ellipse which has (0,5) and (0,-5) as coordinates of its foci and 20 as the sum of its focal radii.

3. Determine the equation of the ellipse which has (0,4) and (,-4) as coordinates of its foci and 16 as the sum of its focal radii.

4. Determine the equation of the ellipse which has (7,0) and (-7,0) as coordinates of its foci and 18 as the sum of its focal radii.

HIGHER ORDER ASSESSMENT TASK

5. Demonstrate the procedure for deriving the equation of the ellipse which has (c,0) and (-c,0) as coordinates of its foci and 2a as the sum of its focal radii.

PERFORMANCE OBJECTIVE VIII-7

Given an equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$, sketch the graph of the relation.

1. Sketch the graph of $\frac{x^2}{4} + \frac{y^2}{9} = 1.$
2. Sketch the graph of $\frac{x^2}{[illegible]} + \frac{y^2}{4} = 1.$
3. Sketch the graph of $\frac{x^2}{5} + \frac{y^2}{4} = 1.$
4. Sketch the graph of $\frac{x^2}{16} + \frac{y^2}{36} = 1$

PERFORMANCE OBJECTIVE VIII-8

Given the coordinates of the foci and the absolute value of the difference of the focal radii, determine the equation of the hyperbola.

1. Determine the equation of the hyperbola which has (0,6) and (0,-6) as coordinates of its foci, and 6 as the absolute value of the difference of the focal radii.

Determine the equation of the hyperbola which has (0,7) and (0,-7) as coordinates of its foci, and 12 as the absolute value of the difference of the focal radii.

3. Determine the equation of the hyperbola which has (5,0) and (-5,0) as coordinates of its foci and 8 as the absolute value of the difference of the focal radii.

4. Determine the equation of the hyperbola which has (3,0) and (-3,0) as coordinates of its foci and 4 as the absolute value of the difference of its focal radii.

HIGHER ORDER ASSESSMENT TASK

1. Demonstrate the procedure for deriving the equation of the hyperbola which has (c,0) and (-c,0) as coordinates of its foci and 2a as the absolute value of the difference of its focal radii.

PERFORMANCE OBJECTIVE VIII-9

Given an equation in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, sketch the graph of the hyperbola.

1. Sketch the graph of $\frac{x^2}{4} - \frac{y^2}{9} = 1$.
2. Sketch the graph of $\frac{y^2}{16} - \frac{x^2}{4} = 1$.
3. Sketch the graph of $\frac{x^2}{36} - \frac{y^2}{4} = 1$.
4. Sketch the graph of $\frac{y^2}{4} - \frac{x^2}{64} = 1$.

PERFORMANCE OBJECTIVE VIII-10

Given a quadratic equation in the form $Ax^2 + By^2 + Cx + Dy + E = 0$:

a) Identify the conic.

b) Write the equation in standard form.

c) State the appropriate basic properties (for the circle, its radius and center; for the parabola, its vertex and axis of symmetry; for the ellipse, its x and y intercepts and the lengths of its major and minor axes; for the hyperbola, its intercepts, and asymptotes).

d) Sketch the graph.

Part A: Match each of the equations 1 - 12 with the name of its graph:

a) line b) circle c) parabola d) ellipse e) hyperbola

1. $2x^2 + 2y^2 + 4y - 9 = 0$
2. $4x - y - 3 = 0$
3. $x^2 - 10x - y + 28 = 0$
4. $x^2 + y^2 - 49 = 0$
5. $x^2 + 6x - y + 28 = 0$
6. $x - \frac{y}{7} = 0$
7. $4x^2 + 25y^2 - 100 = 0$
8. $\frac{x^2}{16} + \frac{y^2}{16} - 1 = 0$
9. $-16x^2 + 9y^2 - 144 = 0$
10. $x^2 - y^2 - 16 = 0$
11. $25x^2 + 9y^2 - 225 = 0$
12. $y^2 - 3y - x + 13 = 0$

PERFORMANCE OBJECTIVE VIII-10 (continued)

13. $4x^2 \quad 3y^2 - 8x - 6y - 5 = 0$

14. $x^2 - 2y^2 - 4x - 6y = 0$

15. $2x^2 + 2y^2 + 4y - 4x - 9 = 0$

Part B

For numbers 3, 7, 8, 9, and 12 of Part A:

a) Write the equation in standard form.

b) State the basic properties (for the circle, its radius and center; for the parabola, its vertex and axis of symmetry; for the ellipse, its x and y intercepts and the lengths of its major and minor axes; for the hyperbola, its intercepts and asymptotes).

c) Sketch the graph.

For each higher order assessment task (13, 14, and 15):

a) Write the equation in standard form.

b) State the basic properties (for the circle, its radius and center; for the ellipse, its center, lengths of its major and minor axes, coordinates of the endpoints of major and minor axes; for the hyperbola, its center, the lengths of its transverse and conjugate axes, the coordinates of the endpoints of the transverse and conjugate axes).

c) Sketch the graph.

PERFORMANCE OBJECTIVE VIII-11

Graph the subsets of the plane defined by a quadratic inequality.

1. Graph the inequality $y > x^2 - 4$.
2. Graph the inequality $9x^2 + 4y^2 \leq 36$.
3. Graph the inequality $(x - 3)^2 + (y + 2)^2 > 25$.
4. Graph the inequality $x^2 - 4y^2 \geq 16$.

PERFORMANCE OBJECTIVE VIII-12

Sketch a hyperbola in the form $xy = k$, where $k \neq 0$.

1. Sketch the function $xy = 6$.
2. Sketch the function $xy = -4$.
3. Sketch the function $xy = \frac{1}{2}$.
4. Sketch the function $xy = \frac{-4}{3}$.

PERFORMANCE OBJECTIVE VIII-13

Given a narrative problem involving inverse variation, translate to an equation and solve.

1. If x varies inversely as y, and x = 3 when y is 8, find y when x is 4.
2. If y varies inversely as x, and y is 3 when x is 9, find x when y is 9.
3. The rates of two meshed gear wheels are inversely proportional to the diameter of the wheels. If one wheel has a diameter of 6 inches and revolves at 750 r.p.m., at what rate does the other wheel revolve, if it has a diameter of 10 inches?
4. The amount of light falling on a given spot is inversely proportional to the square of the distance between the spot and the source of the light. If the spot is 8 lumens when the source is 30 feet away, how much light falls on the spot when the source of the light is 15 feet away?

PERFORMANCE OBJECTIVE VIII-14

Given a narrative problem involving joint or combined variations, translate into an equation and solve.

1. If x varies jointly as y and as the square of z, and $x = 48$ when $y = 3$ and $z = 2$, determine x when $y = 6$ and $z = 3$.
2. If x varies directly as the square of y and inversely as z, and if $x = 2$ when $y = 3$ and $z = 27$, determine x when $y = 2$ and $z = 24$.
3. The pressure (p) needed to force water through a pipe varies directly as the square of the velocity (v) and inversely as the radius (r) of the pipe. If the pressure is 75 pounds when the velocity is 5 and the radius is 2, determine the pressure when the velocity is 6 and the radius is 4.
4. The volume (v) of a right circular cone varies directly as the product of the radius (r) squared and the height (h). If the volume is 432 cu. cm when the radius is 4 cm and the height is 9 cm, determine the radius when the volume is 972 cu. cm and the height remains at 9 cm.

ENRICHMENT

Determine the equation for a conic, given the focus, directrix, and eccentrici

1. Determine the equation of the conic with focus (1,0), directrix $x = 4$, and eccentricity $e = \frac{1}{2}$.

2. Determine the equation of the conic with focus (1,4), directrix $x = 8$, and eccentricity $e = \frac{3}{2}$.

3. Determine the equation of the conic with focus (6,1), directrix $x = 7$, and eccentricity $e = \frac{1}{2}$.

4. Determine the equation of the conic with focus $(0, -\sqrt{3})$, directrix $y = -\frac{\sqrt{3}}{3}$, and eccentricity $e = \sqrt{3}$.

ANSWERS

VIII-1

1. $y = \frac{1}{24} x^2 - 4$
2. $x = -\frac{1}{8}(y + 1)^2$
3. $y = \frac{1}{8}(x - 4)^2 + 2$
4. $y = -\frac{1}{16}(x - 6)^2 + 2$

VIII-2

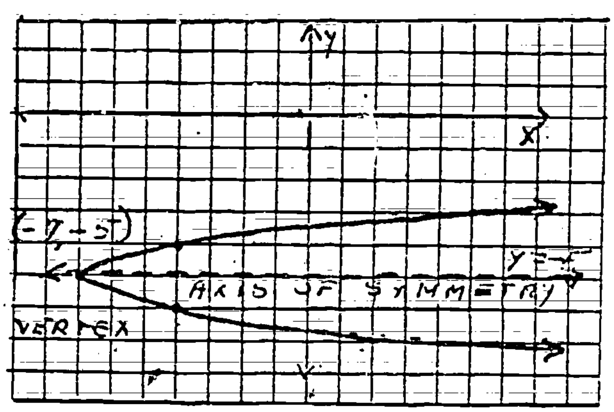
1. a) $x = 3(y + 5)^2 - 7$

   b) $y = -5$

   c) $(-7$

   d) Not a function

   e)



2. a) $f(x) = 3(x - 1)^2 + 5$

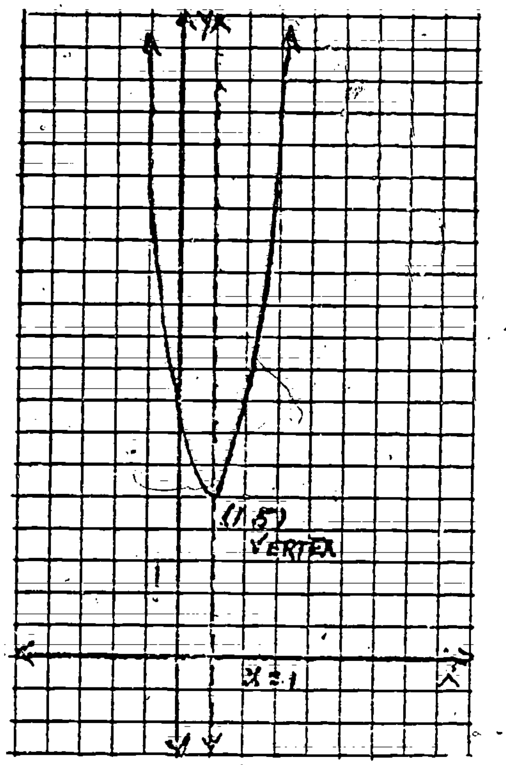
   b) $x = 1$

   c) $(1, 5)$

   d) Minimum value = 1

VIII-2 (continued)

2. e)



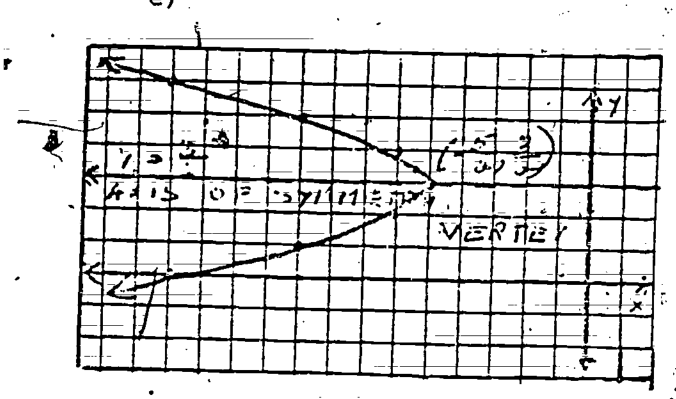
3. a) $x = {}^{-}2(y - \frac{3}{2})^2 - \frac{5}{2}$

b) $y = \frac{3}{2}$

c) $(-\frac{5}{2}, \frac{3}{2})$

d) Not a function

e)



ANSWERS

VIII-2 (continued)

4.

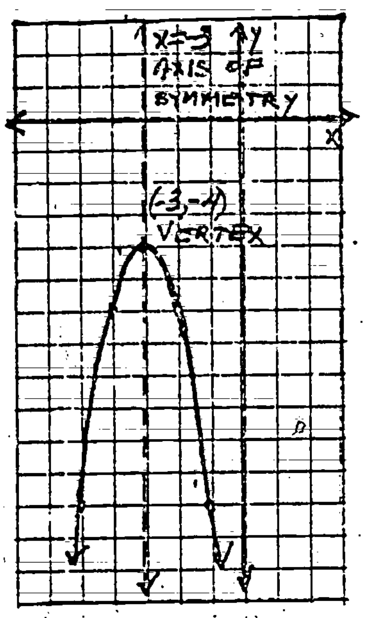
a) $f(x) = {}^{-}2(x + 3)^2 - 4$

b) $x = {}^{-}3$

c) $({}^{-}3, {}^{-}4)$

d) Maximum value = $^{-}4$

e)



VIII-3

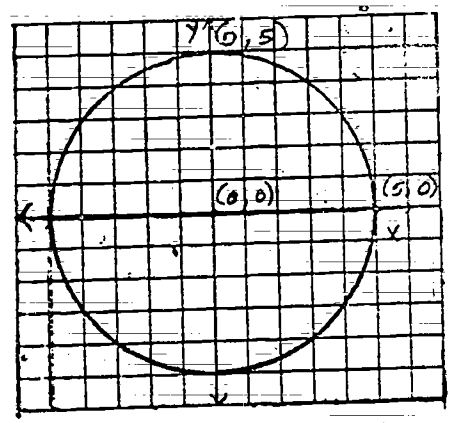
1. 11 stories
2. 20; $^{-}20$
3. 50' x 50'; 2500 sq. ft.
4. \$27.50

VIII-4

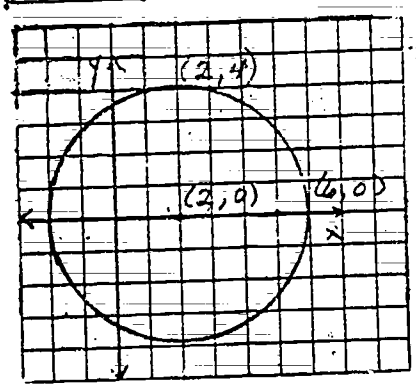
1. $x^2 + y^2 = 36$
2. $x^2 + y^2 = 49$
3. $(x + 5)^2 + (y - 7)^2 = 25$
4. $(x + 1)^2 + (y + 4)^2 = 3$

VIII-5

1.



2.



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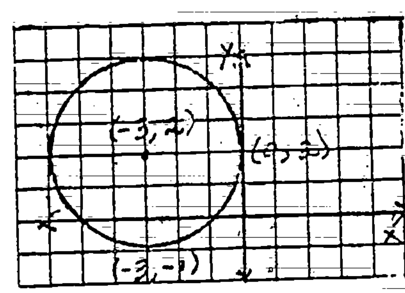

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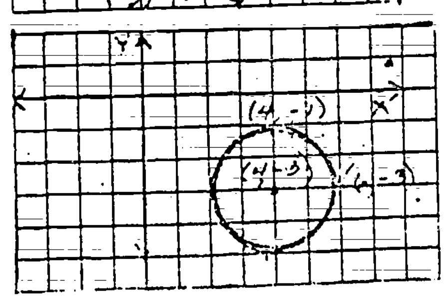


ANSWERS

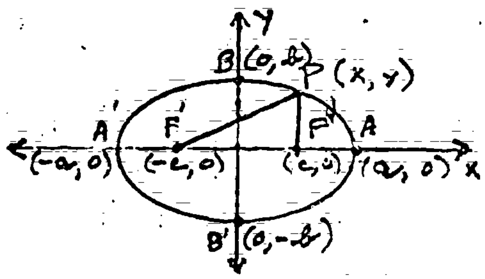
VIII - 6

1. $\frac{x^2}{36} + \frac{y^2}{27} = 1$

2. $\frac{x}{75} + \frac{y}{100} = 1$

3. $\frac{x^2}{48} + \frac{y^2}{64} = 1$

4. $\frac{x^2}{81} + \frac{y^2}{32} = 1$

5. 

$F'P + FP = 2a$

$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$

$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$

$(x+c)^2 + y^2 =$

$4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$

$x^2 + 2xc + c^2 + y^2$

$4a^2 - 4a\sqrt{(x-c)^2 + y^2} + x^2 - 2xc + c^2 + y^2$

$4xc - 4a^2 = -4a\sqrt{(x-c)^2 + y^2}$

$a^2 - xc = a\sqrt{(x-c)^2 + y^2}$

$a^4 - 2a^2xc + x^2c^2 = a^2(x^2 - 2xc + c^2 + y^2)$

$a^4 - 2a^2xc + x^2c^2 = a^2x^2 - 2a^2xc + a^2c^2 + a^2y^2$

$a^4 - a^2c^2 = a^2x^2 - x^2c^2 + a^2y^2$

$a^2(a^2 - c^2) = x^2(a^2 - c^2) + a^2y^2$

$a^2b^2 = b^2x^2 + a^2y^2$

$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

NOTE:

$a > c$

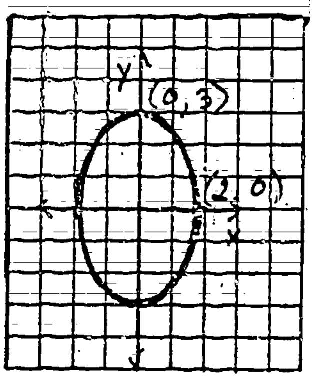
$a^2 > c^2$

$a^2 - c^2 > 0$

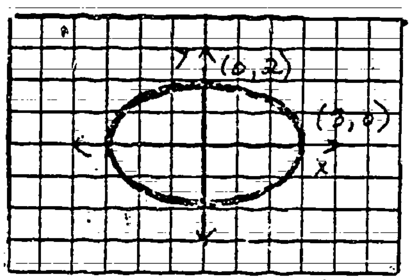
let $a^2 - c^2 = b^2$

VIII-7

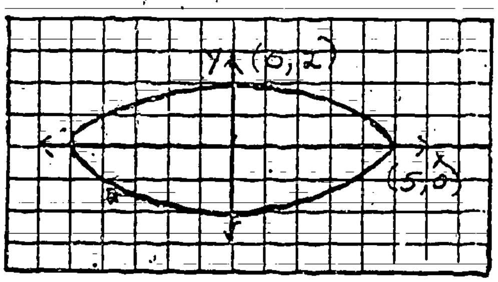
1.



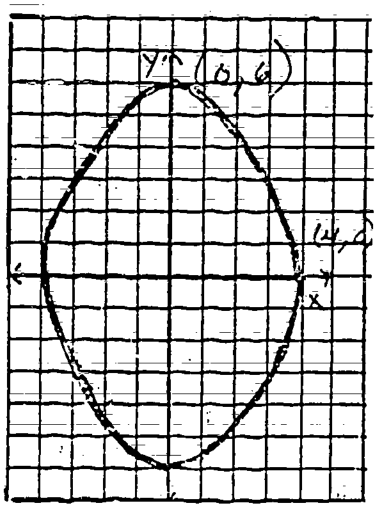
2.



3.



4.



ANSWERS

VIII - 8

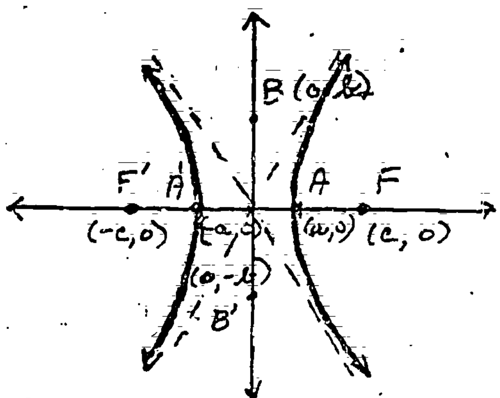
1. $\frac{y^2}{9} - \frac{x^2}{27} = 1$

2. $\frac{y^2}{36} - \frac{x^2}{13} = 1$

3. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

4. $\frac{x^2}{4} - \frac{y^2}{5} = 1$

5.



$|PF' - PF| = 2a$

$\sqrt{(x + c)^2 + y^2} = \sqrt{(x-c)^2 + y^2} = 2a$

$\sqrt{(x + c^2 + y^2} = 2a + \sqrt{(x - c)^2 + y^2}$

$(x + c)^2 + y^2 = 4a^2 + 4a\sqrt{(x - c)^2} + (x-c)^2 + y^2$

$x^2 + 2xc + c^2 + y^2 = 4a^2 + 4a\sqrt{(x - c)^2} + y^2 + x^2 - 2xc + c^2 + y^2$

$4xc - 4a^2 = 4a\sqrt{(x - c)^2 + y^2}$

$xc - a^2 = a\sqrt{(x - c)^2 + y^2}$

$x^2c^2 - 2xca^2 + a^4 = a^2(x^2 - 2xc + c^2 + y^2)$

$x^2c^2 - 2xca^2 + a^4 = a^2x^2 - 2xca^2 + a^2c^2 + a^2y^2$

$x^2c^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4$

$x^2(c^2 - a^2) - a^2y^2 = a^2(c^2 - a^2)$

$x^2b^2 - a^2y^2 = a^2b^2$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

NOTE:

$c > a$

$c^2 > a$

$c^2 - a^2 > 0$

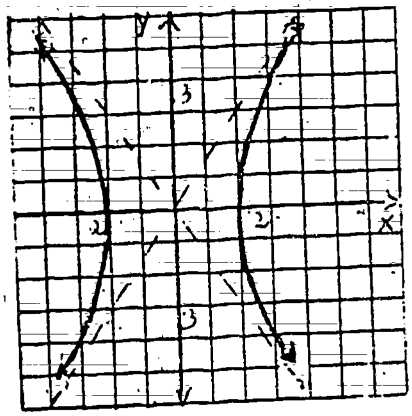
let

$c^2 - a^2 = b^2$

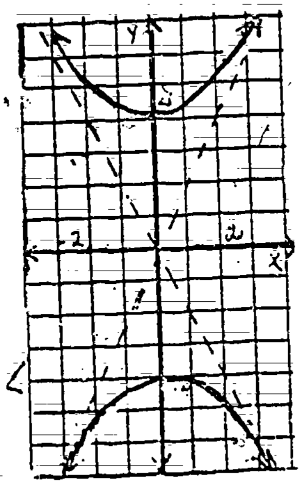
ANSWERS

VIII - 9

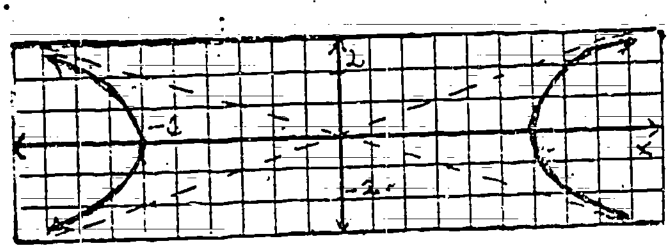
1.



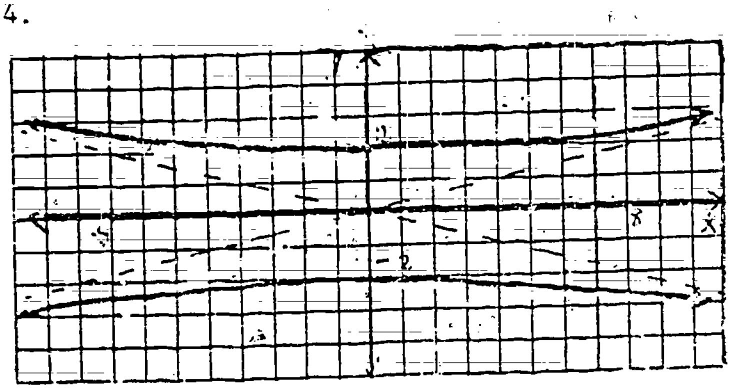
2.



3.



4.



VIII-10

Part A

1. b)
2. a)
3. c)
4. b)
5. c)
6. a)
7. d)
8. b)
9. e)
10. e)
11. d)
12. c)

ANSWERS

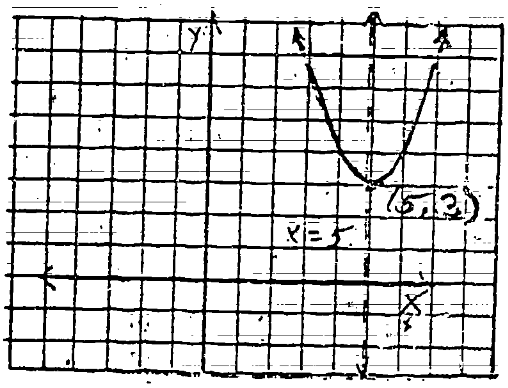
VIII-10 (continued)

Part B

3. a) $y = (x - 5)^2 + 3$

b) vertex (5, 3) axis of symmetry $x = 5$

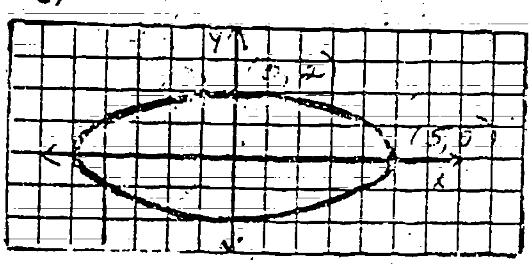
c)



7. a) $\frac{x^2}{25} + \frac{y^2}{4} = 1$

b) (5, 0); (-5, 0); (0, -2); (0, 2) major axis 10, minor axis 8

c)

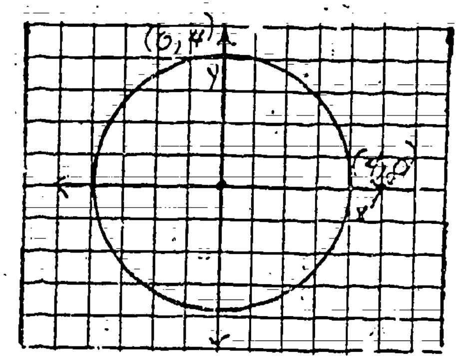


8.

a) $x^2 + y^2 = 16$

b) (0, 0); radius 4

c)

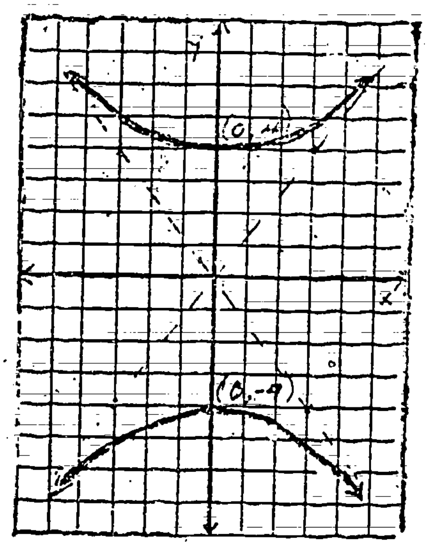


9.

a) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

b) (0, 4); (0, -4); asymptotes $y = \frac{4}{3}x$ and $y = \frac{4}{3}x$

c)

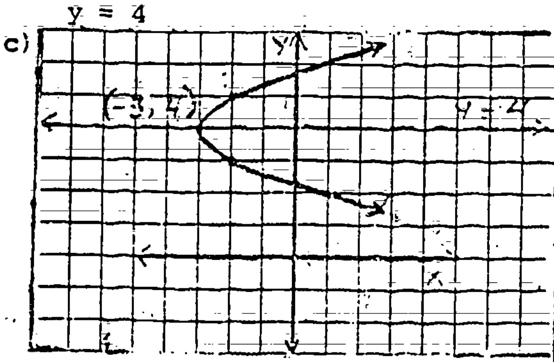


12.

a) $x = (y - 4)^2 - 3$

b) vertex (-3, 4) axis of symmetry $y = 4$

c)



ANSWERS

VIII-10B (continued)

13.

a) $\dfrac{(y-1)^2}{4} + \dfrac{(x-1)^2}{3} = 1$

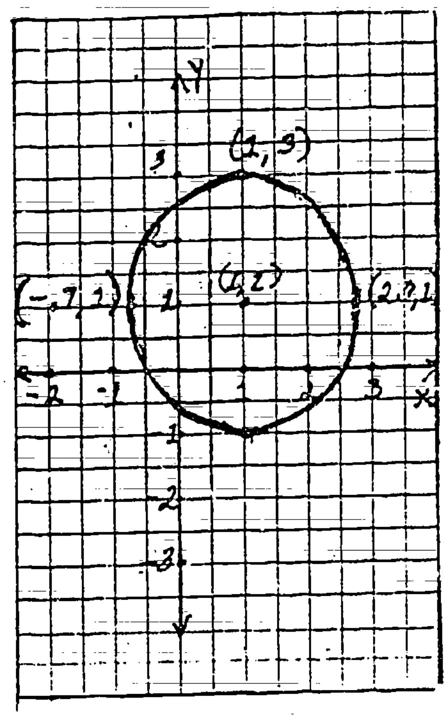
b) center (1, 1); length major axis 4;

length of minor axis $2\sqrt{3}$;

end pts major axis (1, $^-1$), (1, 3)

end pts. minor axis (2.7, 1) ($^-.7$, 1)

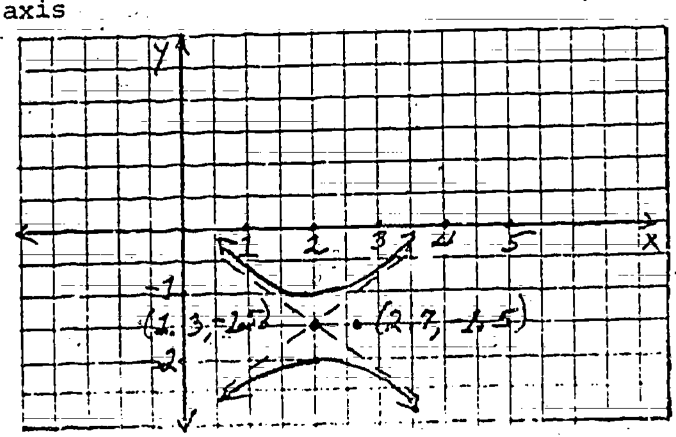
c)



14.

a) $\dfrac{(y+\frac{3}{2})^2}{\frac{1}{4}} - \dfrac{(x-2)}{\frac{1}{2}} = 1$

b) center (2, $^-1.5$); length transverse axis 1; length of conjugate axis $\sqrt{2}$; end pts. transverse axis (2, $^-1$), (2, $^-2$); end pts. conjugate axis (2.7, $^-1.5$), (1.3, $^-1.5$)



ANSWERS

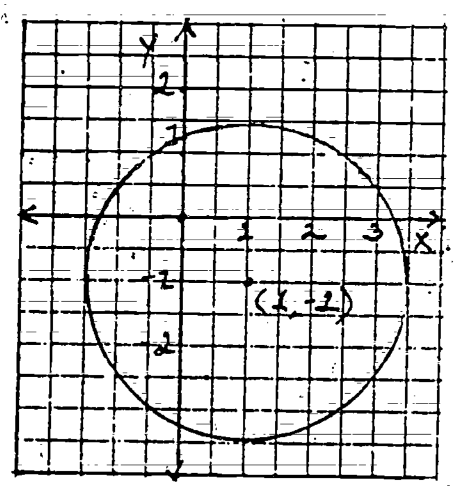
VIII-10B (Continued)

15.

a) $(x - 1)^2 + (y + 1)^2 = \frac{13}{2}$

b) center $(1, ^-1)$; radius $\frac{\sqrt{26}}{2}$

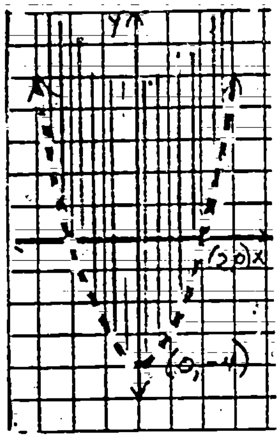
c)



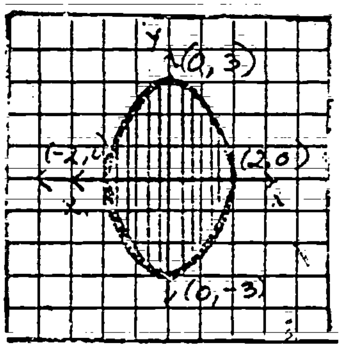
ANSWERS

VIII-11

1.



2.



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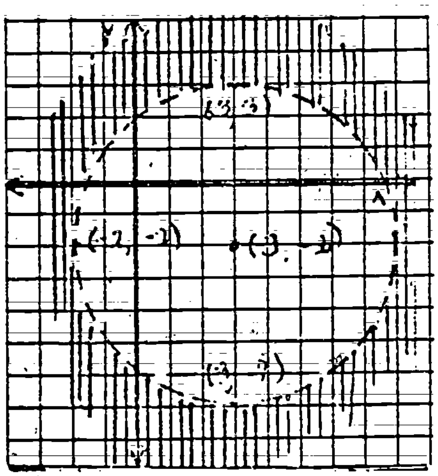

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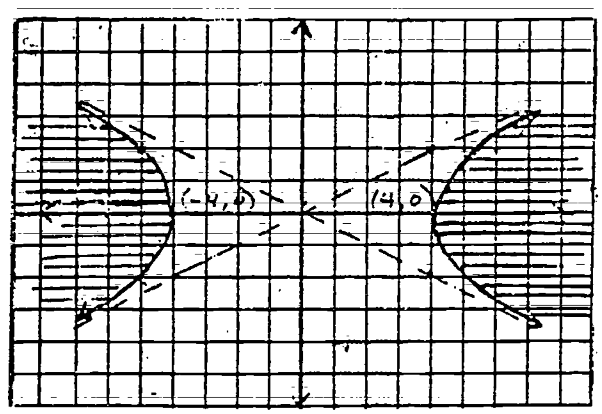
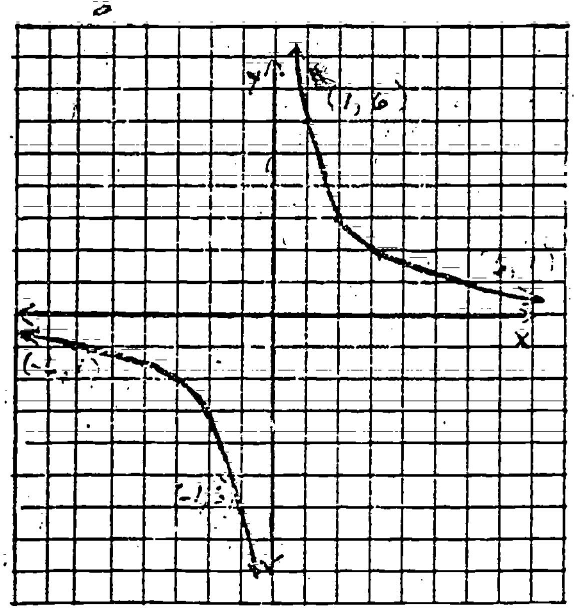


VIII-12

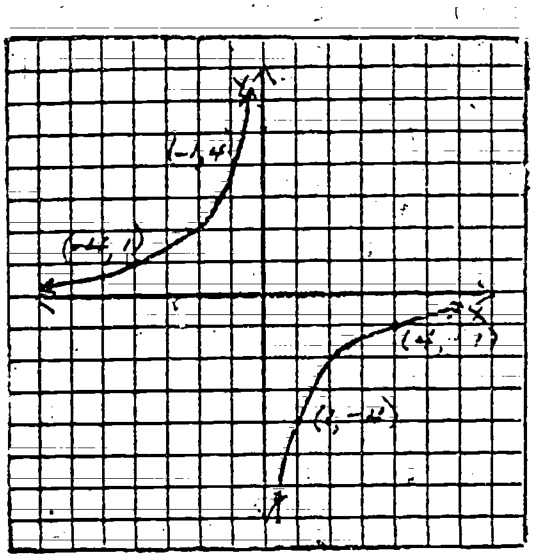
1.



ANSWERS

VIII-12 (continued)

2.



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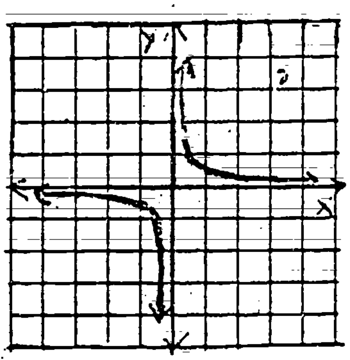

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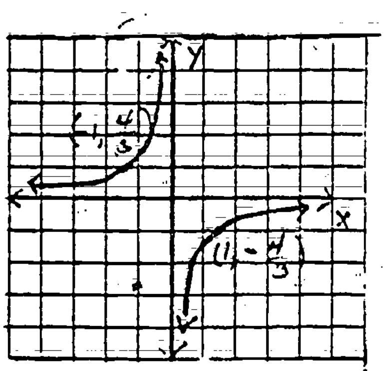


VIII-13

1. $xy = 24 = 4y$; 6
2. $x^2y = 243 = x^2 9$; $3\sqrt{3}$
3. (6) $750 = 4500 = 10x$; 450 r.p.m.
4. $8(30)^2 = 7200 = (15)^2 x$; 32 lumens

VIII-14

1. $x = 216$
2. $x = 1$
3. $p = 54$
4. $6 = r$

Enrichment

1. $3x^2 + 4y^2 - 12 = 0$ or

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

2. $5x^2 - 4y^2 - 136x + 32y + 506 = 0$
3. $3x^2 + 4y^2 = 34x - 8y + 99 = 0$
4. $2y^2 = x^2 - 2 = 0$ or

$$y^2 - \frac{x^2}{2} = 1$$

UNIT IX - SYSTEMS OF OPEN SENTENCES

PURPOSE

This unit extends the concepts and procedures learned in Algebra 1 to solve problems that contain systems of open sentences.

OVERVIEW

This unit develops the student's skill in solving linear and quadratic open sentences, using graphical and algebraic methods.

Some performance objectives contain more than four assessment measures. Those items that best suit instructional needs may be selected.

SUGGESTIONS TO THE TEACHER

Computer Applications: BASIC BASIC, Coan, pp. 182-186; Algebra 2 and Trigonometry, Dolciani (1978), pp. 123, 135; Algebra Two with Trigonometry, Foster, p. 79; Computer Programming in the BASIC Language, Golden, pp. 96, 170, 211 (#42), 213 (#55); Algebra Two and Trigonometry, Keedy, pp. 123, 155; and Algebra Two with Trigonometry, Payne, pp. 522-524.

The time allocation for this unit is 15 days.

VOCABULARY

system of equations
linear-quadratic systems of equations
quadratic-quadratic systems of equations
ordered triple
octant

ENTERING PERFORMANCE OBJECTIVES

1. Graph a pair of linear equations to determine the solution set.

2. Solve a system of linear equations in two variables using the addition method.

3. Solve a system of linear equations in two variables using the substitution method.

UNIT IX - SYSTEMS OF OPEN SENTENCES

DIAGNOSTIC TEST KEYED TO ENTERING PERFORMANCE OBJECTIVES

1. Graph the following pair of equations on the same Cartesian coordinate system: $2x + y = 6$

   $x - y = -3$

   Write the point of intersection as an ordered pair.

2. Use the addition method to solve the system of equations:

   $4x + 3y = -2$

   $x - 3y = 7$

   Write the solution as an ordered pair.

3. Use the substitution method to solve the system of equations:

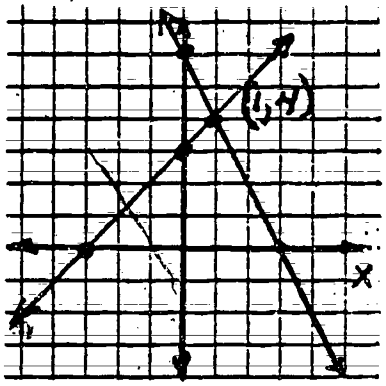
   $2x - 5y = 1$

   $x = 2y + 1$

   Write the solution as an ordered pair.

DIAGNOSTIC TEST

ANSWERS

1. 

2. (1,-2)

3. (3, 1)

UNIT IX - SYSTEMS OF OPEN SENTENCES

PERFORMANCE OBJECTIVES

1. Determine the maximum number of possible intersection points of the graph of linear-quadratic and quadratic-quadratic systems of equations.

2. Determine the solution set of a system of linear-quadratic equations using the substitution method.

3. Determine the solution set of a system of quadratic-quadratic equations in two variables using either the substitution or addition method.

4. Given an ordered triple, construct the coordinate box of the point.

5. Given an equation of a plane, construct that part of the plane that is in the first octant.

6. Determine the solution set of a system of linear equations in three variables.

7. Determine the solution set of a system of linear inequalities in two variables by graphing.

8. Determine the solution of a narrative problem involving a system of open sentences.

9. Graph the subsets of the plane defined by a system of quadratic inequalities.

ENRICHMENT

Use Cramer's Rule to solve a system of linear equations in three variables.

UNIT IX - SYSTEMS OF OPEN SENTENCES

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	324-331	357	368-370	235-237	445 448	378	306-307	373-380	334-336
2	326-328	358-361	370-373	238-239	445-447	381 382	305-310	375-377	337-339
3	329-331	362-364	374-375	238-239	448-449	382	306-310	378-380	340-342
4	104-105 109	141-144	139-142	--	--	172-173	--	--	211-213
5	105	145-150	144-150	--	--	277	--	--	212-213
6	104-110	150-153	150-154	86-87	109-113	277-280	142-145	117-121	215-217
7	111-113 114-117	130-136	126-128	91-92	148-154	273-276	154-161	114-117 127-129	218-220
8	328 331-332	124-128 161-162	[illegible]-124	--	447 450 546	258-259 271-272 280	145-152 310	117; 121 317 381	206-209 343-346
9	324-325	--	--	--	--	383	208-210	373-375	336
ENRICHMENT	560-561 574-576	157-160	155-158	84-86	553-554	286	168-169	525-527	559-560

PERFORMANCE OBJECTIVE IX-1

Determine the maximum number of possible intersection points of the graph of linear-quadratic and quadratic-quadratic systems of equations.

1. The graph of a system of equations consists of a circle and hyperbola. The maximum number of points of intersection is:

   a) 0

   b) 2

   c) 4

   d) An infinite number

   e) None of the above

2. The graph of a system of equations consists of a line and an ellipse. The maximum number of points of intersection is:

   a) 1

   b) 2

   c) 3

   d) An infinite number

   e) None of the above

3. The graph of a system of equations consists of a line and a hyperbola. The maximum number of points of intersections is:

   a) 0

   b) 1

   c) 2

   d) 3

   e) None of the above

4. The graph of a system of equations consists of an ellipse and parabola. The maximum number of points of intersection is:

a) 0

b) 1

c) 2

d) 3

e) None of the above

5. The graph of a system of equations consists of a parabola and a circle. The maximum number of points of intersection is:

a) 3

b) 4

c) 5

d) 6

e) An infinite number

6. The graph of a system of equations consists of a line and circle. The maximum number of points of intersection is:

a) 0

b) 1

c) 2

d) 3

e) None of the above

7. The graph of a system of equations consists of two ellipses. The maximum number of points of intersections is:

   a) 1

   b) 2

   c) 3

   d) 4

   e) None of the above

8. The graph of a system of equations consists of a line and a parabola. The maximum number of points of intersections is:

   a) 0

   b) 1

   c) 2

   d) 3

   e) An infinite number

PERFORMANCE OBJECTIVE IX-2

Determine the solution set of a system of linear-quadratic equations using the substitution method.

1. Use the substitution method to determine the solution set of the system:

   $y = x^2$

   $y = 3x + 4$

2. Use the substitution method to determine the solution set of the system:

   $x = 4y$

   $x^2 + y^2 = 17$

3. Use the substitution method to determine the solution set of the system:

   $2x^2 + y^2 = 24$

   $y = x$

4. Use the substitution method to determine the solution set of the system:

   $x^2 - y^2 = 12$

   $y = 2x$

PERFORMANCE OBJECTIVE IX-3

Determine the solution set of a system of quadratic-quadratic equations in two variables, using either the substitution or addition method.

1. Determine the solution set of the system:

$$2x^2 + y^2 = 9$$
$$x^2 + y^2 = 5$$

2. Determine the solution set of the system:

$$2x^2 + y^2 = 4$$
$$x^2 + y^2 = 2$$

3. Determine the solution set of the system:

$$y^2 - x^2 = 3$$
$$xy = 2$$

4. Determine the solution set of the system:

$$y = x^2 + 3x + 1$$
$$y = 2x^2 - 3$$

5. Determine the solution set of the system:

$$xy = 5$$
$$\frac{1}{x} + \frac{1}{y} = \frac{6}{5}$$

PERFORMANCE OBJECTIVE IX-4

Given an ordered triple, construct the coordinate box of the point.

1. Construct the coordinate box of the point (2, 5, 3).

2. Construct the coordinate box of the point (4, 4, 1).

3. Construct the coordinate box of the point (3, 2, 4).

4. Construct the coordinate box of the point (1, 3, 5).

PERFORMANCE OBJECTIVE IX-5

Given an equation of a plane, construct that part of the plane that is in the first octant.

1. Construct that part of the plane $2x + 3y + 4z = 12$ that is in the first octant.
2. Construct that part of the plane $x + y + z = 4$ that is in the first octant.
3. Construct that part of the plane $3x + y + 3z = 6$ that is in the first octant.
4. Construct that part of the plane $x + 5y + 2z = 5$ that is in the first octant.

PERFORMANCE OBJECTIVE IX-6

Determine the solution set of a system of linear equations in three variables.

1. Determine the solution set of the system:

$$3x + 4y - z = -10$$
$$x + 2y - 5z = -22$$
$$2x - y + 3z = 8.$$

The solution set is:

a) $\{(1, -2, 5)\}$

b) $\{(0, 0, 10)\}$

c) $\{(-2, 0, 4)\}$

d) $\{(4, -8, 2)\}$

e) None of the above

2. Determine the solution set of the system:

$$2x + y + z = 7$$
$$x - y - z = -1$$
$$3x - 4y - 3z = -1$$

The solution set is:

a) $\{(2, -2, 5)\}$

b) $\{(-2, 5, 2)\}$

c) $\{(2, 2, -5)\}$

d) $\{(5, 2, -2)\}$

e) ϕ

f) None of the above

PERFORMANCE OBJECTIVE IX-6 (continued)

3. Determine the solution set of the system:

$$3x + y - z = -4$$

$$2x + y + z = 3$$

$$x - y + 2z = 3$$

The solution set is:

a) $\{(1, 2, 3)\}$

b) $\{(3, 2, 1)\}$

c) $\{(-1, -2, 3)\}$

d) $\{(-1, 2, 3)\}$

e) None of the above

4. Determine the solution set of the system:

$$2x - y + 3z = 10$$

$$3x + 2y - 4z = -8$$

$$x + y - 3z = 5$$

The solution set is:

a) $\{(-5, \frac{-69}{2}, -\frac{23}{2})\}$

b) $\{(5, \frac{69}{2}, \frac{23}{2})\}$

c) $\{(-5, \frac{69}{2}, -\frac{23}{2})\}$

d) ϕ

e) None of the above

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PERFORMANCE OBJECTIVE IX-7

Determine the solution set of a system of linear inequalities in two variables by graphing.

1. Determine the solution set of:

$$2x - y \leq 2$$

$$y \leq 2x$$

by graphing.

2. Determine the solution set of:

$$y < -3x + 4$$

$$y > \frac{2}{3}x - 3$$

by graphing.

3. Determine the solution set of:

$$2x + y \leq 2$$

$$-x + 3y \leq -6$$

by graphing.

4. Determine the solut... set of:

$$-x + 2y \geq 4$$

$$2x + 3y < 6$$

by graphing.

5. Determine the solution set of:

$$x \geq 3$$

$$y \geq 2x + 1$$

by graphing.

PERFORMANCE OBJECTIVE IX-7 (continued)

HIGHER ORDER ASSESSMENT TASK

6. Mark Company manufactures two types of auto parts, both of which require two machines. Each machine is available for a maximum of 180 minutes a day. Part A takes four minutes on machine I and three minutes on machine II; it will yield a profit of $8.00. Part B takes five minutes on machine I and six minutes on machine II; it will yield a $12.00 profit. How many of each part should be produced to maximize the profit?

7. Mr. Green plans to make two types of punch for a large party. He has on hand 32 units of concentrate A and 54 units of concentrate B. Each liter of type I punch requires four units of A and one unit of B. Each liter of type II punch requires one unit of A and six units of B. Determine the maximum number of liters that Mr. Green can make.

PERFORMANCE OBJECTIVE IX-8

Determine the solution of a narrative problem involving a system of open sentences.

1. Determine all pairs of integers for which the sum of their squares is 25 and the difference of their squares is 7.
2. A rectangular lot which lies with its longest side on a river's edge has an area of 72 square meters. If the owner wishes to fence the other three sides, he will need 25 meters of fencing. Determine the dimensions of the lot.
3. Determine all pairs of integers for which the product is 12 and the sum of their squares is 25.
4. Determine the lengths of legs of a right triangle whose hypotenuse is 13 centimeters and whose area is 30 square centimeters.
5. Determine the measures of the angles of a triangle if twice the first, plus the second, is equal to the third, and if the second plus the third is 20 more than three times the first.
6. Beatrice is five times as old as Dick. Ten years from now Beatrice will be three times as old as Dick will be. Find the age of each.
7. A shopkeeper sells a blend of coffee for $4.25 a pound. When mixing the blend he adds together coffee beans selling for $3.50 a pound and beans selling for $4.75 a pound. If he wants to prepare 20 pounds of blend, how many pounds of each kind of coffee must be mixed?

8. Ralph paddled his kayak up the creek to his brother's igloo in four hours. A storm caused Ralph to be left up the creek without a paddle; but using a make-shift oar, he returned downstream to his starting point in $2\frac{1}{2}$ hours. If the round trip was 24 kilometers, find Ralph's average rate and the rate of the current.

9. Mr. Jenkins finished his bus route and counted the change in the coin box. He found a total of 3200 coins worth $398.00. If the coins were dimes and quarters, how many of each coin did Mr. Jenkins have?

PERFORMANCE OBJECTIVE IX-9

> Graph the subsets of the plane defined by a system of inequalities.

1. Graph the following system of inequalities:

$$x^2 + (y - 1)^2 \leq 4$$
$$y \geq \frac{1}{2}x^2 - 1$$

2. Graph the following system of inequalities:

$$x^2 + y^2 - 16 < 0$$
$$2x^2 + 8y^2 > 32$$

3. Graph the following system of inequalities:

$$y^2 - 6y + 7 + x \leq 0$$
$$y^2 - 9x^2 \leq 9$$

4. Graph the following system of inequalities:

$$y \leq x + 2$$
$$25x^2 + 4y^2 - 100 \leq 0$$
$$4y^2 - x^2 \geq 4$$

ENRICHMENT

Use Cramer's Rule to solve a system of linear equations in three variables.

1. Use Cramer's Rule to solve the system:

$$-x + 5y + z = -5$$
$$2x + 3y - 5z = 7$$
$$x + 4y + 3z = 9.$$

2. Use Cramer's Rule to solve the system:

$$-x - 2y + z = 4$$
$$2x - y + 3z = 7$$
$$3x + 5y - z = -4$$

3. Use Cramer's Rule to solve the system:

$$x + 4y + z = 6$$
$$-3x + y + 6z = -3$$
$$2x + y - z = 3$$

4. Use Cramer's Rule to solve the system:

$$3x + y + 3z = 1$$
$$2y + 5z = -3$$
$$-6x - 2y + z = 5$$

ANSWERS

IX-1

1. c)
2. b)
3. c)
4. e)
5. b)
6. c)
7. e)
8. c)

IX-2

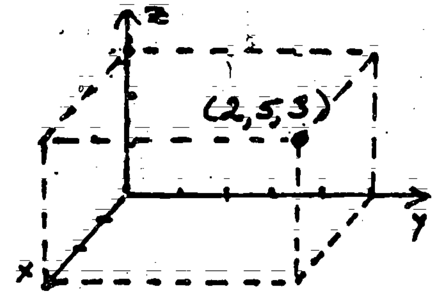
1. $\{(4, 16), (-1, 1)\}$
2. $\{(4, 1), (-4, -1)\}$
3. $\{(2\sqrt{2}, 2\sqrt{2}), (-2\sqrt{2}, -2\sqrt{2})\}$
4. $\{(2i, 4i), (-2i, -4i)\}$

IX-3

1. $\{(x, y): (2, 1), (-2, -1), (-2, 1), (-2, -1)\}$
2. $\{(x, y): (\sqrt{2}, 0), (-\sqrt{2}, 0)\}$
3. $\{(x, y): (1,2), (-1,-2), (2i,-i), (-2i,i)\}$
4. $\{(x, y): (4,29), (-1,-1)\}$
5. $\{(x, y): (5,1), (1,5)\}$

IX-4

1.



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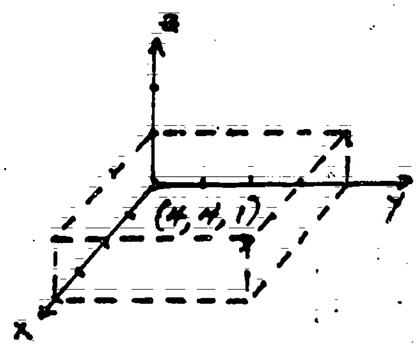

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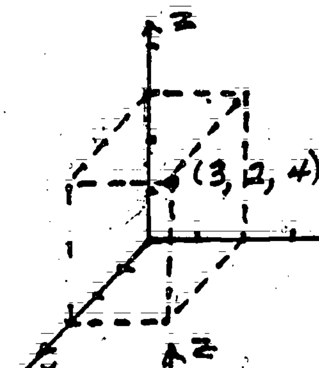

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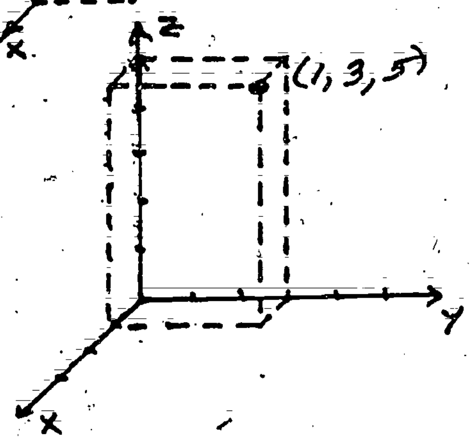
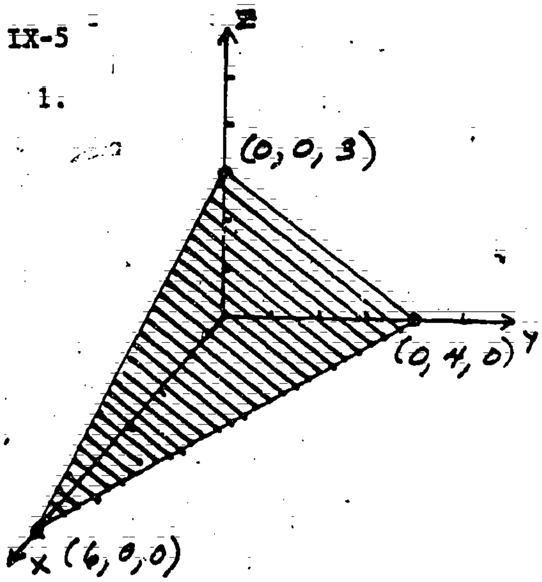


ANSWERS

IX-5

1.



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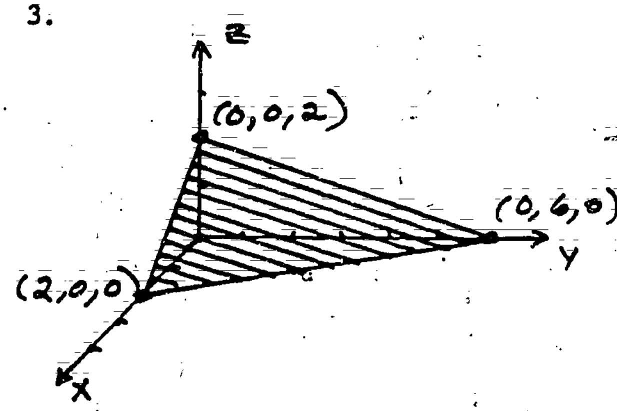

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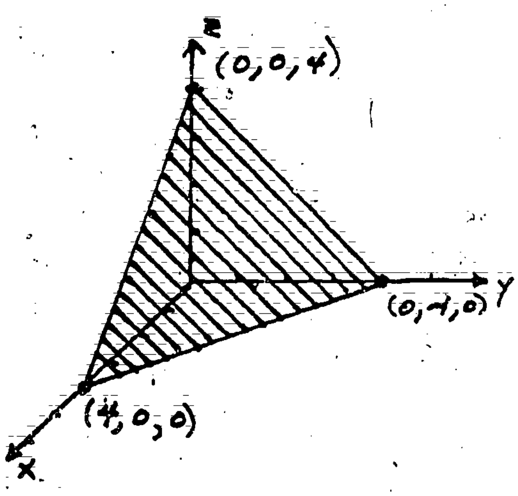

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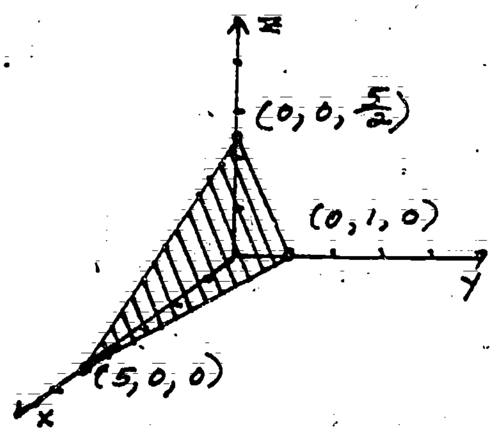


ANSWERS

IX-6

1. c)

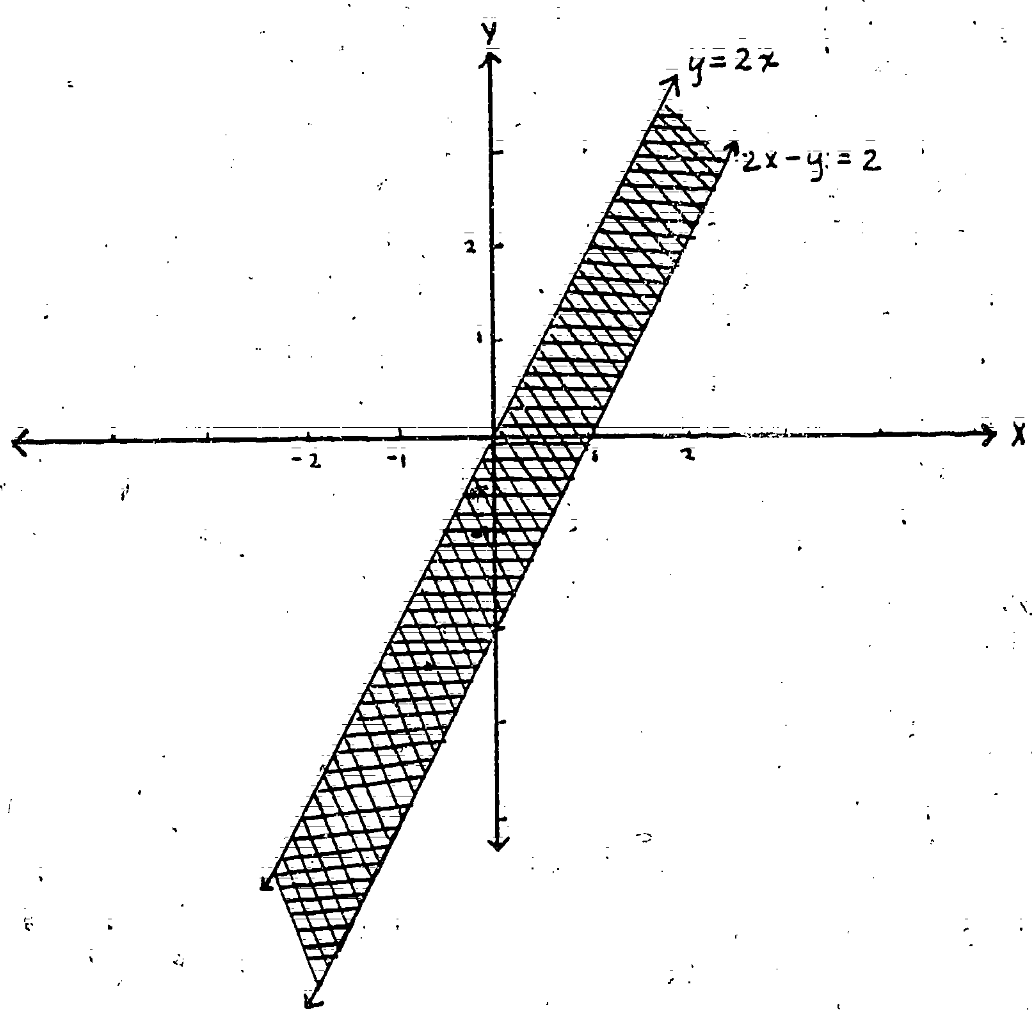
2. a)

3. d)

4. e)

IX-7

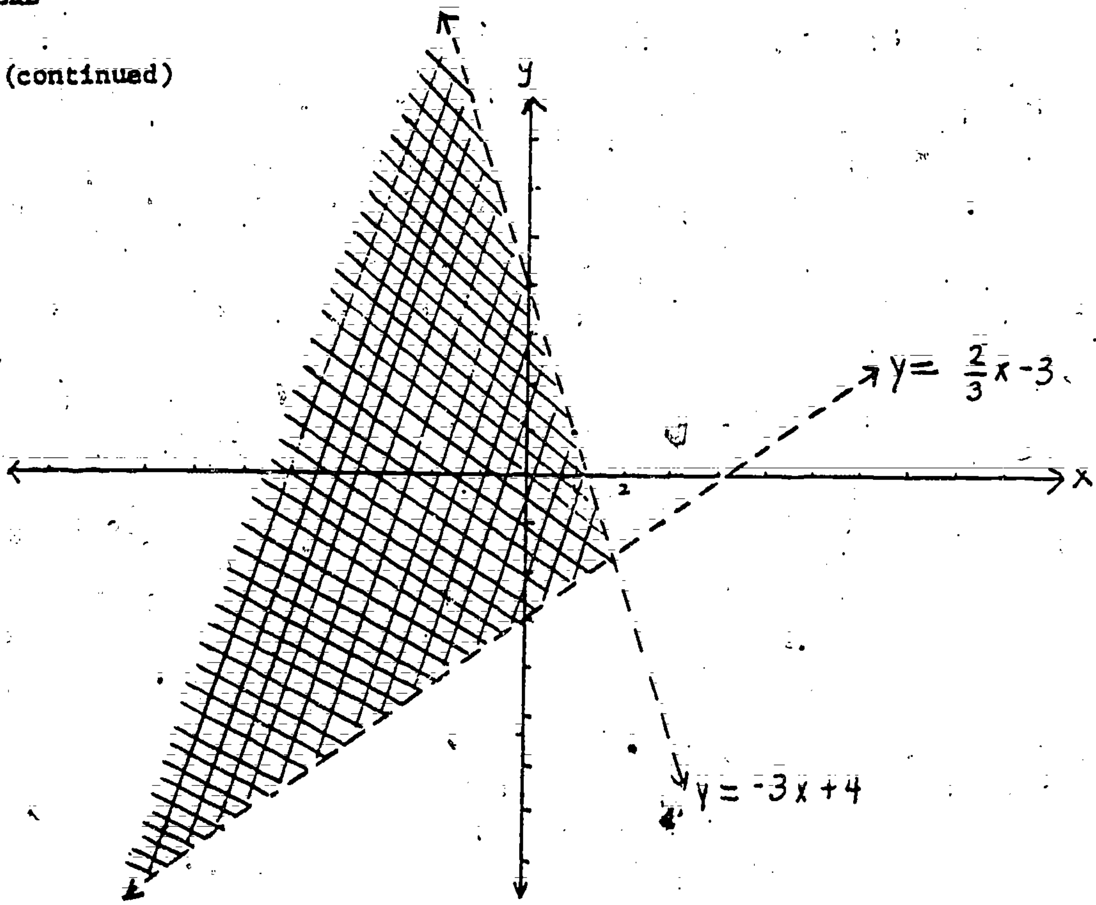
1.



ANSWERS

IX-7 (continued)

2.



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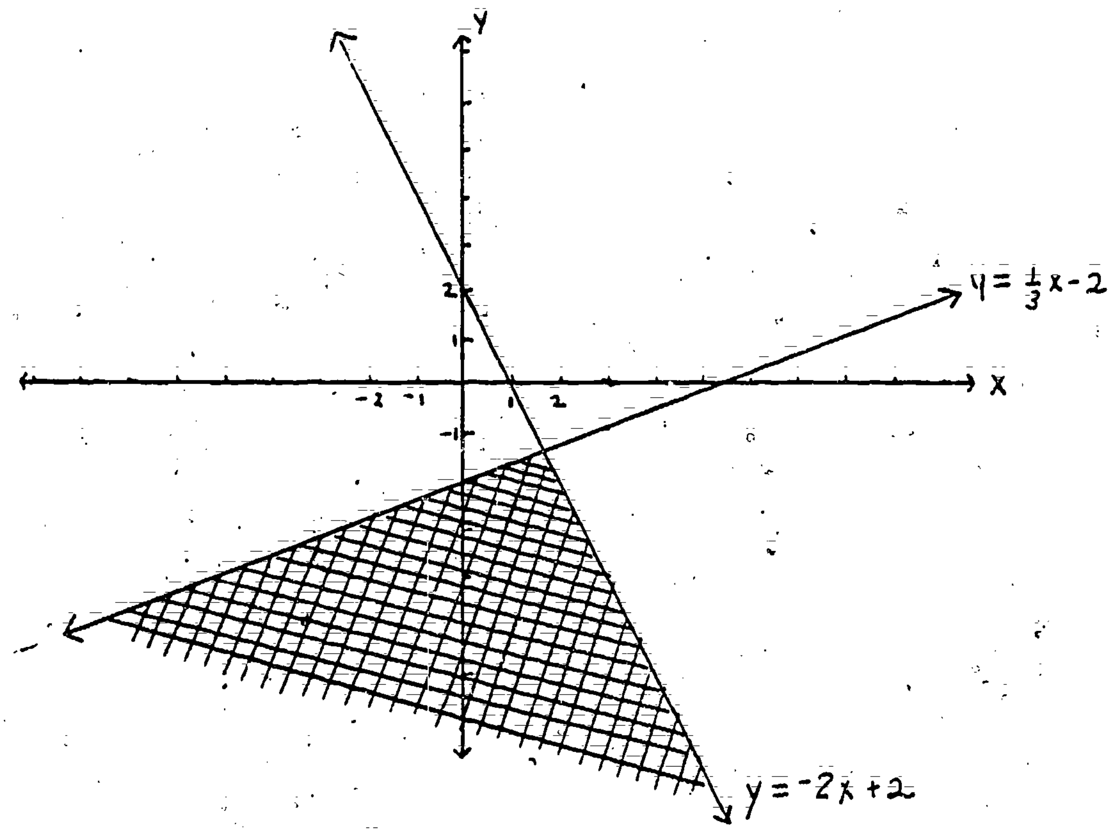
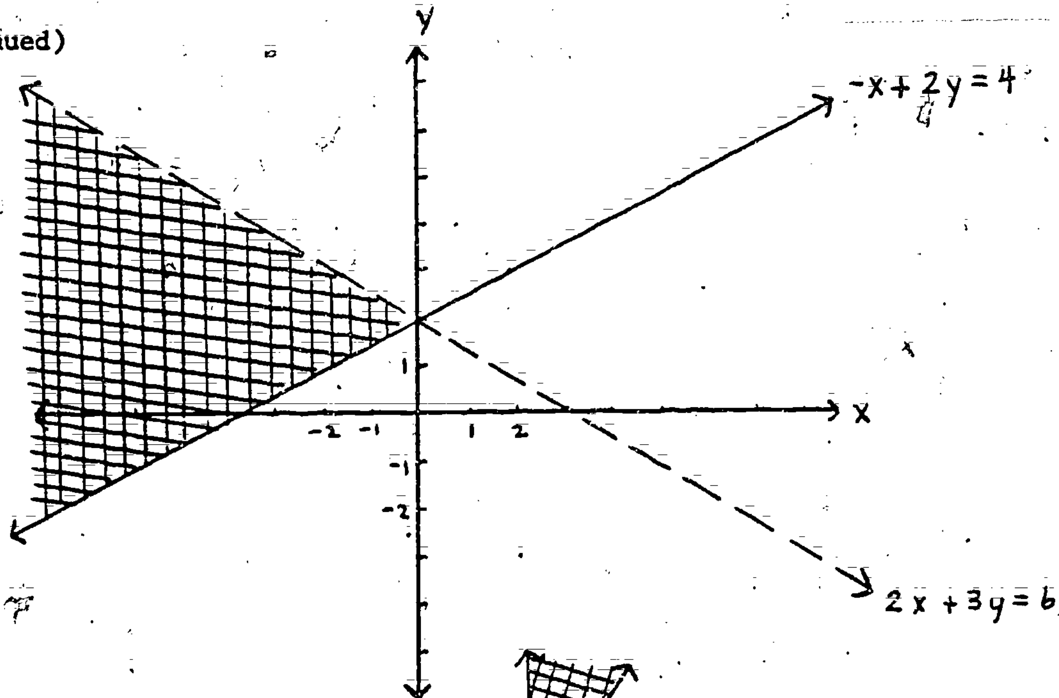


ANSWERS

IX-7 (continued)

4.



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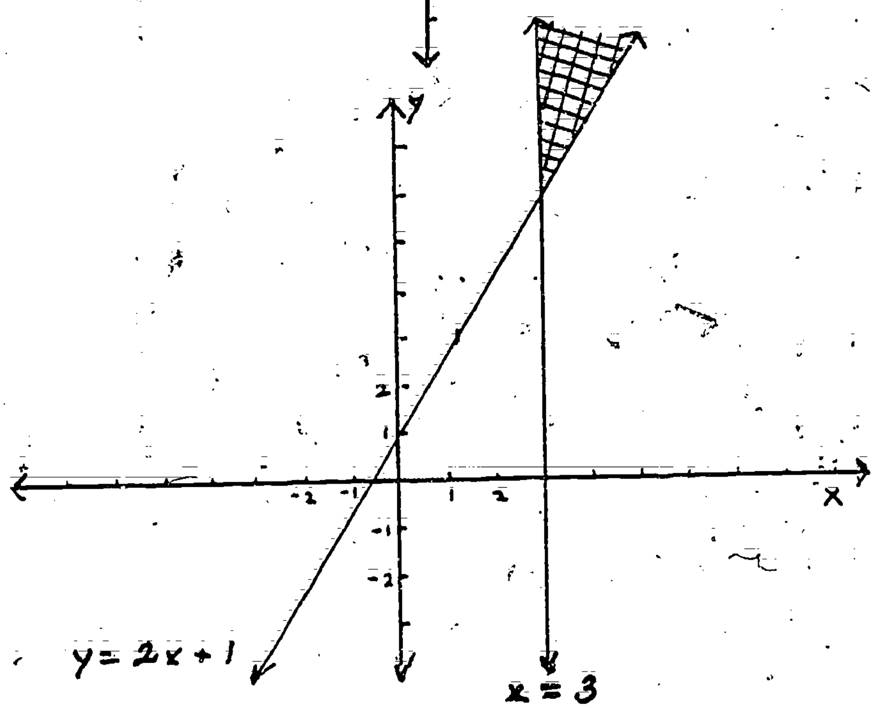


HIGHER ORDER ASSESSMENT TASK

6. (20, 20)

7. 14 liters

ANSWERS

IX-8

1. {(4, 3), (4, -3), (-4, 3), (-4, -3)}
2. {(9 meters by 8 meters), or (16 meters by $\frac{9}{2}$ or $4\frac{1}{2}$ meters)}
3. {(-3, -4), (3, 4)}
4. {5 centimeters and 12 centimeters}
5. (40, 40, 110)
6. Dirk--10 years old

   Beatrice--50 years old
7. 8 lbs of $3.50 coffee

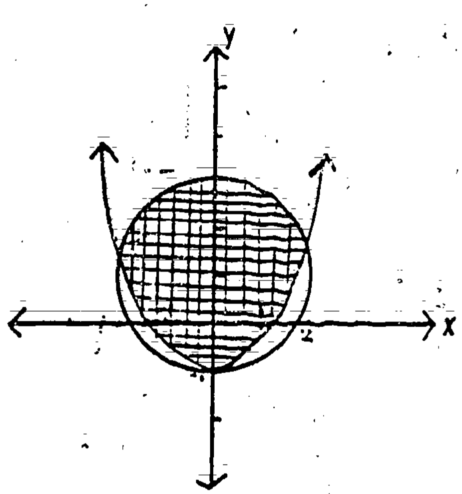
   12 lbs of $4.75 coffee
8. Ralph's rate--3.9 kph

   Current's rate-- .9 kph
9. 2680 dimes

   520 quarters

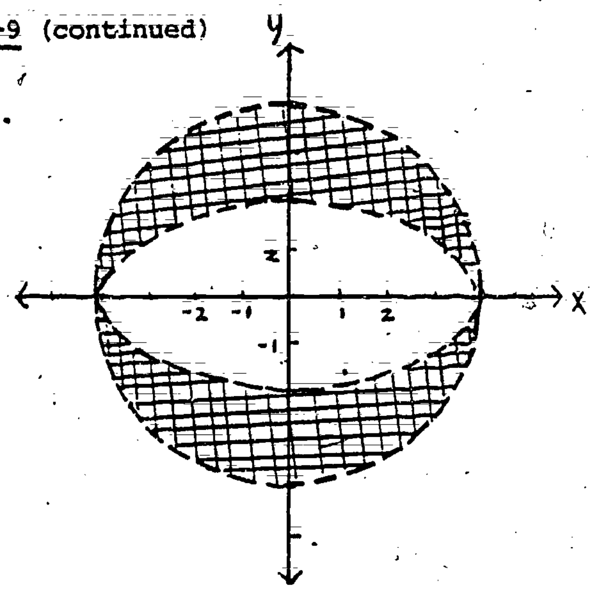
IX-9

1.



IX-9 (continued)

2.



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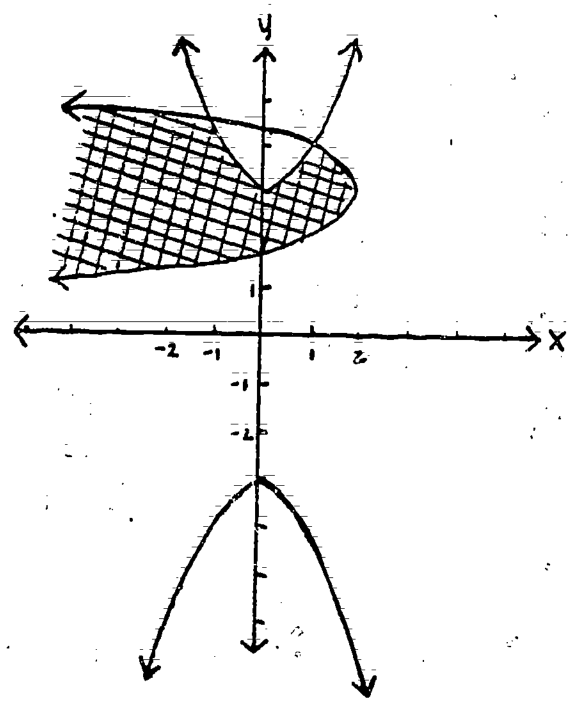
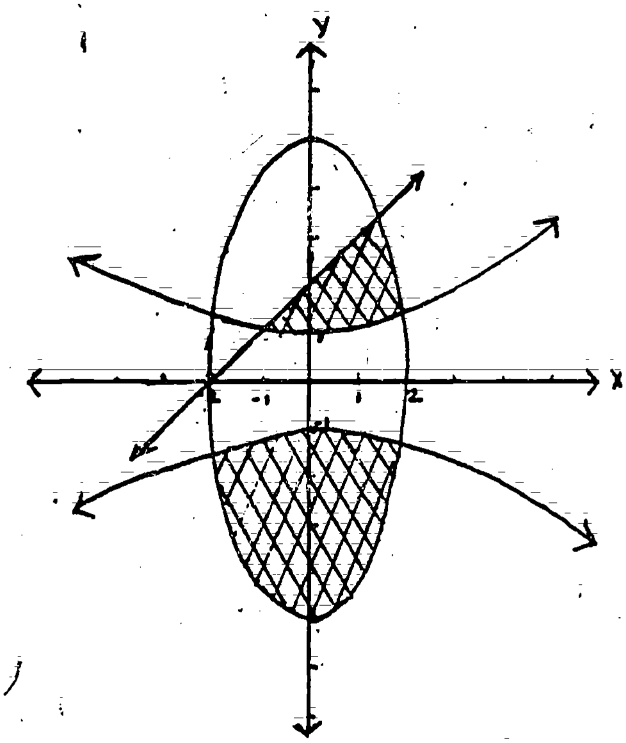


ANSWERS

IX-9 (continued)

4.



ENRICHMENT

1. $\{(6, 0, 1)\}$
2. $\{(-3, 2, 5)\}$
3. $\{(\frac{1}{2}, \frac{3}{2}, -\frac{1}{2})\}$
4. $\{(\frac{2}{3}, -4, 1)\}$

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UNIT X - REAL NUMBER EXPONENTS

PURPOSE

Real number exponents provide further study of expressions and equations as background for the exponential and logarithmic functions in Unit XI.

OVERVIEW

This unit advances the study of exponents by supplementing whole number exponents with integral, rational, and irrational exponents. Although new material is introduced, the student should find the topics closely related to first semester work, and, therefore, a break in the otherwise rigorous second semester.

SUGGESTIONS TO THE TEACHER

This unit will progress rapidly if the student demonstrates mastery of the entering performance objective.

The basic negative exponent equivalencies: $a^{-m} = \frac{1}{a^m}, \frac{1}{a^{-n}} = a^n \ (a \neq 0)$ are not individual performance objectives, but should be emphasized.

Computer Applications: Algebra 2 and Trigonometry, Dolciani (1978), p. 397; Computer Programming in the BASIC Language, Golden, pp. 50-51, 58, 137-138; and Algebra Two and Trigonometry, Keedy, p. 335.

The time allocated for this unit is twelve days.

VOCABULARY

base
power

UNIT X - REAL NUMBER EXPONENTS

PERFORMANCE OBJECTIVES

1. Given an expression involving:

   a) a product of rational expressions
   b) a quotient of rational expressions
   c) a power of a rational expression

   with integral exponents, use the properties of exponents to determine an equivalent expression.

2. Given an expression with positive integral exponents, rename quotients as products using properties of exponents.

3. Given an expression with integral exponents, write an equivalent expression using positive exponents.

4. Write $a^{\frac{m}{n}}$ in radical form, when "a" is a real number and "$\frac{m}{n}$" is a rational number.

5. Determine the equivalent exponential form for a given radical expression.

6. Given an expression in the form $\sqrt[n]{a^m}$, write it in simplest form.

7. Given a number which can be expressed in radical form as $\sqrt[n]{a^m}$, write the radical with a smaller index by reducing m/n.

8. Given an indicated product of radical expressions with different indices, demonstrate the procedure for simplifying to a single radical expression.

9. Given a radical equation in one variable, determine the solution set.

10. Given an expression with irrational exponents, write it in simplest form.

ENRICHMENT

1. Simplify a given radical expression.

2. Given an exponential equation in one variable, determine the solution set.

UNIT X - REAL NUMBER EXPONENTS

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	159-162	174-177	381-384	304-306	311-315 328-330	44-45 59 294-295	67-72	209-216	115-120
2	159-162	--	171-175	304-306	327-329	300-301	72	213-216	119-123
3	339-340	--	171-175	304-306	298-302	296 301	72	389-390	119-123
4	339-341	367-369	381-383	311-313	328-330	302-305	247-249	389-391	351-353
5	339-341	368-369	381-383	311-313	328-330	304-305	248	389-391	351-353
6	339-341	--	381-383	311-313	330	304	--	389-391	351-353
7	339-341	--	381-383	311-313	330	307	253-254	389-391	351-353
8	339-341	368-369	381-383	311-313	330	307	253-254	389-391	351-353
9	251-255	370	282-284	321-323	331-334	309-313	257-260	272-274	296-298
10	342-344	372	--	314-315	--	--	--	390-393	354-357

UNIT X - REAL NUMBER EXPONENTS

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
ENRICHMENT									
1	341	370	--	--	--	315	255	391	--
2	343-344	372	384-387	--	333	314-315	--	394	355-357

X-4

PERFORMANCE OBJECTIVE X-1

Given an expression involving

a) a product of rational expressions

b) a quotient of rational expressions

c) a power of a rational expression

with integral exponents, use the properties of exponents to determine an equivalent expression.

1. Using the properties of exponents, simplify each of the following:

a) $x^2 \cdot x^{-4}$

b) $\frac{x^3}{x^{-3}}$

c) $(xy)^{-2}$

d) $(\frac{x}{y^{-1}})^2$

e) $(x\, y^{-1})^{-3}$

(NOTE: 4 out of 5 for mastery)

2. Using the properties of exponents, simplify each of the following:

a) $5a^3 \cdot 6a^{-4}$

b) $\frac{3a^{-2}}{2a^3}$

c) $(a^2b)^{-3}$

d) $(\frac{a}{b^2})^{-1}$

e) $(a^{-2}b^{-1})^2$

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE X-1 (continued)

3. Using the properties of exponents, simplify each of the following:

a) $3x^2 \cdot 4x^{-2}$

b) $\frac{a^{-3}}{a}$

c) $(4x^{-1})^2$

d) $(\frac{3a^{-1}}{2b})^2$

e) $(a^2b^3c^{-1})^2$

(NOTE: 4 of 5 for mastery)

4. Using the properties of exponents, simplify each of the following:

a) $x^a \cdot x^2$

b) $\frac{x^3}{x^a}$

c) $(2x^4)^a$

d) $(\frac{x^2}{y})^a$

e) $(x^ay^a + b)^2$

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE X-2

Given an expression with positive integral exponents, rename quotients as products using properties of exponents.

1. Write $\frac{x^2}{y^3}$ as a product not in fraction form.

2. Write $\frac{3x^2z}{y^3w}$ as a product not in fraction form.

3. Write $\frac{2}{a^3} + \frac{5}{b^2}$ as a sum of products not in fraction form.

4. Write $5x^2 + x - 6 + \frac{2}{x} - \frac{1}{x^2}$ as a sum of products not in fraction form.

PERFORMANCE OBJECTIVE X-3

Given an expression with integral exponents, write an equivalent expression using positive exponents.

1. Write $\frac{a^{-4}b^{-2}}{c^{-2}}$ as an expression with positive exponents.

2. Write $\frac{3^{-1}x^2y^{-4}}{zw^{-3}}$ as an expression with positive exponents.

3. Write $a^{-2}b + a^3b^{-2}$ as an expression with positive exponents.

4. Write $\frac{(a + b)^{-2}}{2^{-1}c^2b^{-3}}$ as an expression with positive exponents.

PERFORMANCE OBJECTIVE X-4

Write $a^{\frac{m}{n}}$ in radical form, when "a" is a real number and "$\frac{m}{n}$" is a rational number.

1. Determine the equivalent radical form for $7^{-\frac{1}{2}}$.

2. Determine the equivalent radical form for $2ab^{\frac{1}{2}}$.

3. Determine the equivalent radical form for $3x^{\frac{2}{3}}y^{\frac{1}{3}}$.

4. Determine the equivalent radical form for $a^{\frac{1}{3}}c^{\frac{1}{2}}$.

5. Determine the equivalent radical form for $x^{-\frac{2}{5}}y^{-\frac{2}{5}}$.

6. Determine the equivalent radical form for $5^{\frac{1}{4}}a^{\frac{3}{4}}b^{\frac{1}{4}}$.

PERFORMANCE OBJECTIVE X-5

Determine the equivalent exponential form for a given radical expression.

1. Determine the equivalent exponential form for $\sqrt[3]{x^2y}$.

2. Determine the equivalent exponential form for $a\sqrt[5]{b^3c^2}$.

3. Determine the equivalent exponential form for $5\sqrt[3]{x} \cdot \sqrt[4]{y^3}$.

4. Determine the equivalent exponential form for $\sqrt[3]{8x - y}$.

PERFORMANCE OBJECTIVE X-6

Given an expression in the form $\sqrt[n]{a^m}$, write it in simplest form.

1. Write $\sqrt[5]{32^3}$ in simplest form.

2. Write $\sqrt{\left(\frac{1}{4}\right)^5}$ in simplest form.

3. Write $\sqrt[4]{x^{12}}$ in simplest form.

4. Write $\sqrt[5]{1024^3}$ in simplest form.

PERFORMANCE OBJECTIVE X-7

Given a number which can be expressed in radical form as $\sqrt[n]{a^m}$, write the radical with a smaller index by reducing $\frac{m}{n}$.

1. Write $\sqrt[4]{25}$ as another radical with a smaller index.

2. Write $\sqrt[6]{9x^4}$ as another radical with a smaller index.

3. Write $\sqrt[6]{144x^2}$ as another radical with a smaller index.

4. Write $3\sqrt[4]{64a^2}$ as another radical with a smaller index.

PERFORMANCE OBJECTIVE X-8

Given an indicated product of radical expressions with different indices, demonstrate the procedure for simplifying to a single radical expression.

1. Demonstrate the procedure for simplifying $(\sqrt{2})(\sqrt[3]{2})$ to a single radical expression.

2. Demonstrate the procedure for simplifying $(\sqrt{3})(\sqrt[3]{2})$ to a single radical expression.

3. Demonstrate the procedure for simplifying $(\sqrt{3x})(\sqrt[3]{3y^2})$ to a single radical expression.

4. Demonstrate the procedure for simplifying $(\sqrt[4]{2x^3})(\sqrt[3]{3y^2})$ to a single radical expression.

PERFORMANCE OBJECTIVE X-9

Given a radical equation in one variable, determine the solution set.

1. Determine the solution set for x if $\sqrt{5x - 7} = \sqrt{2x + 5}$.

2. Determine the solution set for x if $\sqrt{x - 4} + 3 = 0$.

3. Determine the solution set for x if $x^{\frac{2}{3}} - 5 = 20$.

4. Determine the solution set for x if $\sqrt{3x - 6} + 2 = x$.

5. Determine the solution set for x if $2\sqrt{x} + \sqrt{2x - 1} = 0$.

PERFORMANCE OBJECTIVE X-10

Given an expression with irrational exponents, write it in simplest form.

1. Simplify $\left(2^{\sqrt[3]{2}}\right)^{\sqrt[3]{4}}$.

2. Simplify $\left(\frac{5^{\sqrt{2}}}{2^{\sqrt{8}}}\right)^{\sqrt{2}}$.

3. Simplify $\frac{2^{\sqrt{3}}}{2^{\sqrt{2}}}$.

4. Simplify $\left(3^{\sqrt{2}-1}\right)\left(3^{\sqrt{2}+3}\right)$.

5. Simplify $\left(3^{\sqrt{2}} + 2^{\sqrt{3}}\right)\left(3^{\sqrt{2}} - 2^{\sqrt{3}}\right)$.

ENRICHMENT

1. Simplify given radical expressions.

   a) Simplify $\sqrt{\sqrt{\sqrt{512}}}$.

   b) Simplify $\sqrt[3]{\frac{3^{2.1}}{3^{0.3}}}$.

2. Given an exponential equation in one variable, determine the solution set.

   a) $8^{n} = 2^{n+3}$

   b) $\left(\frac{1}{2}\right)^{n-1} = 32^{3n-1}$

ANSWERS

X-1

1. a) x^{-2} or $\frac{1}{x^2}$

   b) x^6

   c) $x^{-2}y^{-2}$ or $\frac{1}{x^2y^2}$

   d) $\frac{x^2}{y^{-2}}$ or x^2y^2

   e) $x^{-6}y^3$ or $\frac{y^3}{x^6}$

2. a) $30a^{-1}$ or $\frac{30}{a}$

   b) $\frac{3}{2a^5}$ or $\frac{3a^{-5}}{2}$

   c) $a^{-6}b^{-5}$ or $\frac{1}{a^6b^5}$

   d) $\frac{a^{-1}}{b^{-2}}$ or $\frac{b^2}{a}$

   e) $a^{-4}b^{-2}$ or $\frac{1}{a^4b^2}$

3. a) 12

   b) a^{-4} or $\frac{1}{a^4}$

   c) $16x^{-2}$ or $\frac{16}{x^2}$

   d) $\frac{9a^{-2}}{4b^2}$ or $\frac{9}{4a^2b^2}$

   e) $a^4b^6c^{-2}$ or $\frac{a^4b^6}{c^2}$

4. a) x^{a+2}

   b) x^{3-a}

   c) 2^ax^{4a}

   d) $\frac{x^{2a}}{y^{-a}}$ or $x^{2a}y^a$

   e) $x^{2a}y^{2(a+b)}$ or $x^{2a}y^{2a+2b}$

X-2

1. x^2y^{-3}
2. $3w^{-1}x^2y^{-3}z$
3. $2a^{-3} + 5b^{-2}$
4. $5x^2 + x - 6 + 2x^{-1} - x^{-2}$

X-3

1. $\frac{c^2}{a^4b^2}$
2. $\frac{w^3x^2}{3y^4z}$
3. $\frac{b}{a^2} + \frac{a^3}{b^2}$
4. $\frac{2b^3}{c^2(a+b)^2}$

X-4

1. $\frac{1}{\sqrt{7}}$
2. $2a\sqrt{b}$
3. $3\sqrt[3]{x^2y}$
4. $\sqrt[3]{a}\sqrt{c}$
5. $\frac{1}{\sqrt[5]{x^3y^2}}$
6. $4\sqrt{5a^3b}$

UNIT X - REAL NUMBER EXPONENTS

ANSWERS

X-5

1. $x^{\frac{2}{3}} y^{\frac{1}{3}}$ or $(x^2y)^{\frac{1}{3}}$

2. $ab^{\frac{3}{5}}c^{\frac{2}{5}}$, or $a(b^3c^2)^{\frac{1}{5}}$

3. $5x^{\frac{1}{3}}y^{\frac{3}{4}}$

4. $(8x - y)^{\frac{1}{3}}$

X-6

1. 8

2. $\frac{1}{32}$

3. x^3

4. 64

X-7

1. $\sqrt{5}$

2. $\sqrt[3]{3x^2}$

3. $\sqrt[3]{12x}$

4. $6\sqrt{2a}$

X-8

1. $\sqrt[6]{32}$

2. $\sqrt[6]{108}$

3. $\sqrt[6]{243x^3y^4}$

4. $\sqrt[12]{648x^9y^8}$

X-9

1. $\{4\}$

2. ϕ

3. $\{125, -125\}$

4. $\{2, 5\}$

5. ϕ

X-10

1. 4

2. $\frac{25}{16}$

3. $2^{\sqrt{3}-\sqrt{2}}$

4. $3^{2\sqrt{2}+2}$ or $3^{2(\sqrt{2}+1)}$

5. $3^{2\sqrt{2}} - 2^{2\sqrt{3}}$ or $9^{\sqrt{2}} - 4^{\sqrt{3}}$

ENRICHMENT

1. a) $2\sqrt[8]{2}$

   b) $\sqrt[5]{27}$

2. a) $\left\{\frac{3}{2}\right\}$

   b) $\left\{\frac{3}{8}\right\}$

UNIT XI - EXPONENTIAL AND LOGARITHMIC FUNCTIONS

PURPOSE

Logarithms introduce an alternative method for solving difficult problems involving products, quotients, and powers of numbers.

OVERVIEW

This unit begins with the development of exponential functions and their inverses. The student learns to use the table and interpolate. Antilogarithms and the laws of logarithms are also studied.

SUGGESTIONS TO THE TEACHER

The relationship between exponential and logarithmic functions can be seen through the use of their graphs and by an examination of inverse functions.

Some confusion may be eliminated if the study of the laws of logarithms is introduced without encumbering it with interpolation.

Since logarithms are the basis for slide rules, they can be introduced as an enrichment at this time.

The use of calculators for addition, subtraction, multiplication, and division may be used at the discretion of the teacher for selected performance objectives.

Computer Applications: Algebra 2 and Trigonometry, Dolciani (1978), p. 397; Algebra 2 and Trigonometry, Dolciani (1980), pp. 410, 414-415; Computer Programming in the BASIC Language, Golden, pp. 137 (#51, 52), 170 (#39).

The time allocated for this unit is approximately twenty days.

VOCABULARY

common logarithm
characteristic
mantissa
linear interpolation
antilogarithm
scientific notation
exponential function
logarithmic function

UNIT XI - EXPONENTIAL AND LOGARITHMIC FUNCTIONS

ENTERING PERFORMANCE OBJECTIVES

1. State a given number in scientific notation.
2. Given a number in scientific notation, write it in decimal form.

DIAGNOSTIC TEST KEYED TO ENTERING PERFORMANCE OBJECTIVES

1. State in scientific notation: 63,400.
2. State in scientific notation: 722 million.
3. Write in decimal form: 6.91×10^6.
4. Write in decimal form: 7.3×10^{-4}.

ANSWERS.

1. 6.34×10^4
2. 7.22×10^8

6,910,000

4. 0.00073

PERFORMANCE OBJECTIVES

1. Select exponential functions from a list of given functions.

2. Construct the graph of an exponential function with two first degree variables.

3. State the domain and range of exponential functions.

4. Given a logarithmic equation, write an equivalent exponential equation.

5. Given an exponential equation, write an equivalent logarithmic equation.

6. Given logarithmic equations of the form $\log_b N = r$ where any two of the three variables are given, determine the value of the third variable.

7. Write a given problem as to the sum or difference of logarithms, by using either one or both of the laws: $\log_b xy = \log_b x + \log_b y$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

8. Using the law $\log_b x^r = r \cdot \log_b x$, write the equivalent expression for the logarithm of a power of a number.

9. Using the laws of logarithms, state a given problem as the logarithm of a single number.

10. Using tables of common logarithms, name the mantissa of the logarithm of a given number.

11. Determine the characteristic of the common logarithm of a number.

12. Name the mantissa and characteristic of the common logarithm of a number.

13. Given log a, determine $\log (a \times 10^n)$, where n is an integer.

14. Using a table of common logarithms, determine the antilogarithm of a given number.

15. Using the laws of logarithms, determine the logarithmic equation of products, quotients, and powers of numbers.

16. Determine the common logarithms of products, quotients, and powers of numbers.

17. Determine the solution set of a logarithmic equation in one variable.

18. Approximate logarithms and antilogarithms of numbers not given in the tables by using linear interpolation.

19. Use common logarithms to solve narrative problems.

20. Use common logarithms to solve a given equation.

UNIT XI - EXPONENTIAL AND LOGARITHMIC FUNCTIONS

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	342-344	371 375	390-393	--	482-484	--	--	392-394	354-357
2	342-344	371	390-393	331-333	483-484	322-324	357-359	392-394	354-357
3	344-348	--	390-393	331	--	--	358-360 364-365	397-400	358-361
4	346-348	376-377	390-393	328-330	487-489	328-330	365-367	394-397	358-360
5	349	376-377	390-393	328-330	486-489	328-330	365-368	394-397	358-360
6	346-348	377	390-393	331-333	487-489	331	367-368	400-403	358-360
7	354-355	379-381	394-397	335-337	490-494	335-338	369-371	409-412	369-371
8	358-359	393	406-409	335-337	491-494	336-338	370-371	412-415	373-375
9	354-355	381	394-397	335-337	494	337	371	409-415	369-375
10	349-351	382-383	397-399	339-341	495-498	332-334	376-378	404-406	362-364

XI-4

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
11	349-351	382-383	397-399	339-341	495-498	339	375-378	404-406	362-364
12	349-351	382-383	397-399	339-341	495-498	339-341	378	404-406	362-364
13	349-351	382	397-399	339-341	497-498	339-341	375-378	404-406	362-364
14	349-351	383-384	397-399	339-341	499-500	333-334 340-341	378-379	404-406	362-364
15	354-363	394-395	402-406	345-347	501-502	342-344	383-386	409-415	369-375
16	354-363	395-396 398-401	402-406	345-347	501-502	342-344	383-386	409-415	369-375
17	357 363	384-386	411-414	335-337	508	338	371 386-388	395-397 402-403	359-360 371-372 375
18	351-354	384-386	399-402	342-344	503	345-348	380-382	406-408	365-367
19	363-364	394 396-397 401	409-410 414	352-355	509-513	348-350 352	387-388	412-416 419	379-382
20	365-366	398-401	411-413	349-351	507-508	351-352	386-388	417-418	376-378

XI-5

PERFORMANCE OBJECTIVE XI-1

Select exponential functions from a list of given functions.

1. Select the exponential function(s) from the following set:

   $\{y = x^2,\ f(x) = 2^x,\ 3^2 = 9,\ x = 3^y,\ 6^3 = 216,\ a^x = y\}$

2. Select the exponential function(s) from the following set:

   $\{x = y^3,\ y = 2^x,\ 6x^2 = y,\ 6^3 = 216,\ f(x) = 3x\}$

3. Select the exponential function(s) from the following set:

   $\{3x^2 = y,\ f(x) = 5x,\ 4^3 = 64,\ x = 4^y,\ f(x) = 7^x\}$

4. Select the exponential function(s) from the following set:

   $\{f(x) = 2^x,\ 2^5 = 32,\ 2y^2 = 5x,\ f(x) = x^2,\ x = 2^y\}$

PERFORMANCE OBJECTIVE XI-2

Construct the graph of an exponential function with two first degree variables.

1. Construct the graph of $y = 2^x$

2. Construct the graph of $y = 3^x$.

PERFORMANCE OBJECTIVE XI-2 (continued)

3. Construct the graph of $x = 2^y$

4. Construct the graph of $x = 3^y$

PERFORMANCE OBJECTIVE XI-3

State the domain and range of exponential functions.

1. State the domain and range of the exponential function $f(x) = 2^x$.
2. State the domain and range of the exponential function $f(x) = 3^x + 2$.
3. State the domain and range of the exponential function $f(x) = 4^{-x}$.
4. State the domain and range of the exponential function $f(x) = 5^x - 2$.

PERFORMANCE OBJECTIVE XI-4

Given a logarithmic equation, write an equivalent exponential equation.

1. Write an equivalent exponential equation for $\log_3 9 = 2$.
2. Write an equivalent exponential equation for $\log_2 32 = 5$.
3. Write an equivalent exponential equation for $\log_x x^3 = 3$.
4. Write an equivalent exponential equation for $\log\ 10{,}000 = 4$.

PERFORMANCE OBJECTIVE XI-5

Given an exponential equation, write an equivalent logarithmic equation.

1. Write an equivalent logarithmic equation for $8^2 = 64$.

2. Write an equivalent logarithmic equation for $3^5 = 243$.

3. Write an equivalent logarithmic equation for $x^6 = 64$.

4. Write an equivalent logarithmic equation for $x^3 = 729$.

PERFORMANCE OBJECTIVE XI-6

Given logarithmic equations of the form $\log_b N = r$ where any two of the three variables are given, determine the value of the third variable.

1. Determine the solution set for r if $\log_5 625 = r$.

2. Determine the solution set for N if $\log_2 N = -5$.

3. Determine the solution set for b if $\log_b \frac{1}{27} = -3$.

4. Determine the solution set for r if $\log\ 0.00001 = r$.

PERFORMANCE OBJECTIVE XI-7

Write a given problem as the sum or difference of logarithms, by using either one or both of the laws: $\log_b xy = \log_b x + \log_b y$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

1. Using laws of logarithms, write an equivalent expression for $\log\ (17)(43)$.

2. Using laws of logarithms, write an equivalent expression for $\log_5 \frac{2}{3}$.

3. Using laws of logarithms, write an equivalent expression for $\log_5 6$.

4. Using laws of logarithms, write an equivalent expression for $\log\ \frac{21}{11}$.

PERFORMANCE OBJECTIVE XI-8

Using the law $\log_b x^r = r \cdot \log_b x$, write the equivalent expression for the the logarithm of a power of a number.

1. Using laws of logarithms, write an equivalent expression for $\log 3^5$.

2. Using laws of logarithms, write an equivalent expression for $\log_5 49$.

3. Using laws of logarithms, write an equivalent expression for $\log_5 \sqrt{17}$.

4. Using laws of logarithms, write an equivalent expression for $\log\ \frac{9}{25}$.

PERFORMANCE OBJECTIVE XI-9

Using the laws of logarithms, state a given problem as the logarithm of a single number.

1. Select an expression equivalent to log 6 + log 5:

   a) $\log \frac{6}{5}$

   b) log 11

   c) log 30

   d) 30

2. Select an expression equivalent to $\log_5 22 - \log_5 7$:

   a) $\log_5 15$

   b) $\log_5 \frac{22}{7}$

   c) $\log_5 29$

   d) $\frac{22}{7}$

3. Select an expression equivalent to $2 \log_5 5 + 2 \log_5 2$:

   a) $2 \log_5 7$

   b) 100

   c) $\log_5 10 + \log_5 4$

   d) $\log_5 100$

PERFORMANCE OBJECTIVE XI-9. (continued)

4. Select an expression equivalent to $\frac{1}{3}(\log\ 5 - 2\log\ 8)$:

a) $\sqrt[3]{5} - 8^2$

b) $\log\ \sqrt[3]{5} - \log\ 8^2$

c) $\log\ \frac{5}{3} - \log\ \frac{8^2}{3}$

d) $\log\left(\frac{\sqrt[3]{5}}{4}\right)$

PERFORMANCE OBJECTIVE XI-10

Using tables of common logarithms, name the mantissa of the logarithm of a given number.

1. Using a table of common logarithms, name the mantissa of the logarithm of each of the following numbers:

a) 6.31

b) 790

c) 0.083

d) 5

e) 21.1

(NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE XI-10 (continued)

2. Using a table of common logarithms, name the mantissa of the logarithm of each of the following numbers:

   a) 3.46

   b) 270

   c) 0.0923

   d) 5.01

   e) 60.0

   (NOTE: 4 of 5 for mastery)

3. Using a table of common logarithms, name the mantissa of the logarithm of each of the following numbers;

   a) 8.03

   b) 410

   c) 26.7

   d) 30.0

   e) 3

   (NOTE: 4 of 5 for mastery)

4. Using a table of common logarithms, name the mantissa of the logarithm of each of the following numbers:

   a) 4.75

   b) 3.90

   c) 5.61

   d) 80.2

   e) 70.0

   NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE XI-11

Determine the characteristic of the common logarithm of a number.

1. Determine the characteristic of the common logarithm of each of the following:

   a) 563

   b) 0.073

   c) 9.0

   d) 72.60

   e) 0.0000132

   (NOTE: 4 of 5 for mastery)

2. Determine the characteristic of the common logarithm of each of the following:

   a) 0.161

   b) 7

   c) 2640

   d) 6,000,000,000

   e) 0.00079

   (NOTE: 4 of 5 for mastery)

3. Determine the characteristic of the common logarithm of each of the following:

   a) 743,000

   b) 0.00631

   c) 13

   d) 8.34

   e) 0.000000075

   (NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE XI-11 (continued)

4. Determine the characteristic of the common logarithm of each of the following:

   a) 9230

   b) 17

   c) 6.90

   d) 0.00138

   e) 0.5

   (NOTE: 4 of 5 for mastery)

PERFORMANCE OBJECTIVE XI-12

Name the mantissa and characteristic of the common logarithm of a number.

1. When log x = 3.6291, name

   a) the characteristic

   b) the mantissa

2. When log x = 0.1430, name

   a) the characteristic

   b) the mantissa

3. When log x = 2.7853, name

   a) the characteristic

   b) the mantissa

4. When log x = 7.2967-10, name

   a) the characteristic

   b) the mantissa

PERFORMANCE OBJECTIVE XI-13

Given log a, determine log($a \times 10^n$), where n is an integer.

1. If log 4.97 =0.6964, determine log 4970.
2. If log 5.6 =0.7482, determine log 0.00056.
3. If log 1.9 =0.2788, determine log 19.
4. If log 8.08 =0.9074, determine log 0.0808.

PERFORMANCE OBJECTIVE XI-14

Using a table of common logarithms, determine the antilogarithm of a given number.

1. Using a table of logarithms, determine antilog 2.9165.
2. Using a table of logarithms, determine antilog 8.6149-10.
3. Using a table of logarithms, determine antilog 5.3579.
4. Using a table of logarithms, determine antilog 7.8768-10.

PERFORMANCE OBJECTIVE XI-15

> Using the laws of logarithms, determine the logarithmic equation of products, quotients, and powers of numbers.

1. If $C = \frac{(617)(0.073)^3}{\sqrt{85.3}}$, which of the following logarithmic equations would be used to compute log C?

   a) $\log 617 \times 3 \log .073 \div \frac{1}{2} \log 85.3 = \log C$

   b) $\frac{1}{2} \left(\frac{\log 617 + 3 \log .073}{\log 85.3}\right) = \log C$

   c) $\log 617 + 3 \log .073 - \frac{1}{2}\, 85.3 = \log C$

   d) $\frac{\log 617 + 3 \log .073}{\frac{1}{2} \log 85.3} = \log C$

2. If $C = \frac{726}{\sqrt[3]{(51.4)(0.0689)}}$, which of the following logarithmic equations would be used to compute log C?

   a) $\log 726 - \frac{1}{3} \log 51.4 - \frac{1}{3} \log .0689 = \log C$

   b) $\frac{\log 726}{\frac{1}{3}(\log 51.4 \times \log .0689)} = \log C$

   c) $\frac{\log 726}{\frac{1}{3}(\log 51.4 + \log .0689)} = \log C$

   d) $\log 726 - \left(\log \frac{51.4}{3} + \log \frac{.0689}{3}\right) = \log C$

PERFORMANCE OBJECTIVE XI-15 (continued)

3. If $C = \sqrt{\dfrac{1269 \times 523}{697}}$, which of the following logarithmic equations would be used to compute log C?

a) $\frac{1}{2}\left(\dfrac{\log 1269 \times \log 523}{\log 697}\right) = \log C$

b) $\frac{1}{2}(\log 1269 + \log 523 - \log 697) = \log C$

c) $\frac{1}{2}\left(\dfrac{\log 1269 + \log 523}{\log 697}\right) = \log C$

d) $\log \dfrac{1269}{2} + \log \dfrac{523}{2} - \log \dfrac{697}{2} = \log C$

4. If $C = \sqrt{\dfrac{\sqrt[4]{163}}{(52)(.0673)^3}}$, which of the following logarithmic equations would be used to compute log C?

a) $\frac{1}{2}[\frac{1}{4} \log 163 \div (\log 52 \times 3 \log .0673)] = \log C$

b) $\frac{1}{2}\left(\dfrac{\frac{1}{4} \log 163}{\log 52 + 3 \log .0673}\right) = \log C$

c) $\frac{1}{2}[\frac{1}{4} \log 163 - (\log 52 + 3 \log .0673)] = \log C$

d) $\frac{1}{2}(\log \dfrac{163}{4} - \log 52 + 3 \log .0673) = \log C$

PERFORMANCE OBJECTIVE XI-16

Determine the common logarithms of products, quotients, and powers of numbers.

1. Using a table of common logarithms, determine $\log \dfrac{672}{\sqrt{31.8}}$.

2. Using a table of common logarithms, determine $\log \sqrt[3]{\dfrac{7.94}{863}}$.

3. Using a table of common logarithms, determine $\log \dfrac{21^5}{.0892}$.

4. Using a table of common logarithms, determine $\log \sqrt{\dfrac{7.32^2}{(612)(58)}}$.

PERFORMANCE OBJECTIVE XI-17

Determine the solution set of a logarithmic equation in one variable.

1. Determine the solution set for x if $\log_7 x = 2 \log_7 6 - \frac{1}{2} \log_7 16$.

2. Determine the solution set for x if $\log_5 x = 2 \log_5 12 - \frac{1}{3} \log_5 216$.

3. Determine the solution set for x if $\log_3 (x - 2) + \log_3 x = 1$.

4. Determine the solution set for x if $\log \ (x - 4) - \log \ (x - 1) = 1$.

PERFORMANCE OBJECTIVE XI-18

Approximate logarithms and antilogarithms of numbers not given in the tables by using linear interpolation.

1. Use linear interpolation to determine an approximate value for log 2.436.

2. Use linear interpolation to determine an approximate value for antilog 2.6907.

3. Use linear interpolation to determine an approximate value for log 693.2.

4. Use linear interpolation to determine an approximate value for antilog 7.9780 - 10.

PERFORMANCE OBJECTIVE XI-19

Use common logarithms to solve narrative problems.

1. If the volume of a cube is 693 cubic inches, what is the length of a side?

2. If $3,000 is invested at 9% compounded semi-annually, what will it amount to in 5 years?

   (Answer in three significant figures)

3. What principal amount will have a value of $5,000 in ten years, if invested at 10% compounded semi-annually?

   (Answer in four significant figures)

4. How many years will it take $600, invested at 8% compounded semi-annually, to increase to $1,000?

   (Answer in three significant figures)

PERFORMANCE OBJECTIVE XI-20

Use common logarithms to solve a given equation.

1. Determine the solution set for x:

   $\log_3 8 = x$

2. Determine the solution set for x:

   $\log_6 7 = x$

3. Determine the solution set for x:

   $4^{3x+1} = 11$

4. Determine the solution set for x:

   $3^{x+1} = 5^{2x-3}$

HIGHER ORDER ASSESSMENT TASKS

1. The growth of a certain bacteria is found by using the exponential equation $E(t) = 50 \times 16^t$, where t is time in hours and E(t) is the number of bacteria in the culture.

   a) What is the initial amount of bacteria?

   b) How many bacteria are present after $\frac{1}{4}$ hour?

   c) How many bacteria are present after $\frac{1}{2}$ hour?

2. The law of decay of a radioactive element is given by $E(t) = 180 \times 3^{-.02t}$ where t is time in years and E(t) is the amount of substance present at time t.

   a) What is the initial amount of radioactive substance?

   b) How much substance is present after 100 years?

   c) How long will it take the substance to decay to $\frac{1}{3}$ the original amount?

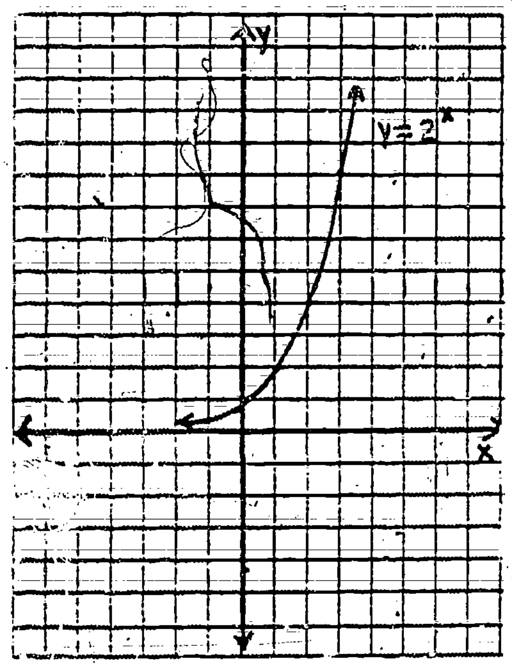
ANSWERS

XI-1

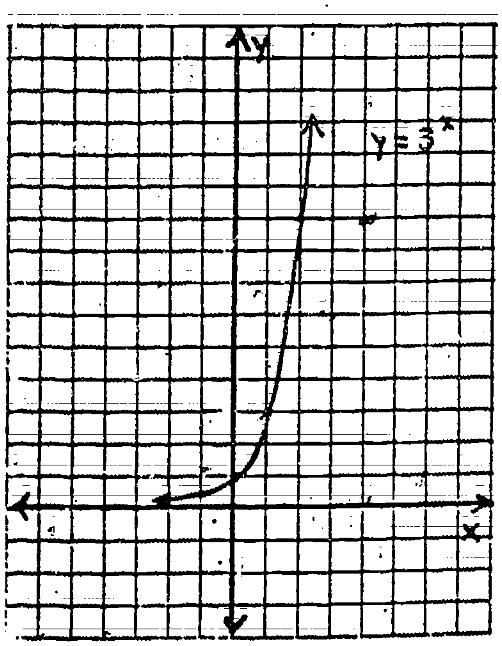
1. $\{f(x) = 2^x,\ x = 3^y,\ a^x = y\}$
2. $\{y = 2^x\}$
3. $\{x = 4^y,\ f(x) = 7^x\}$
4. $\{f(x) = 2^x,\ x = 2^y\}$

XI-2

1.

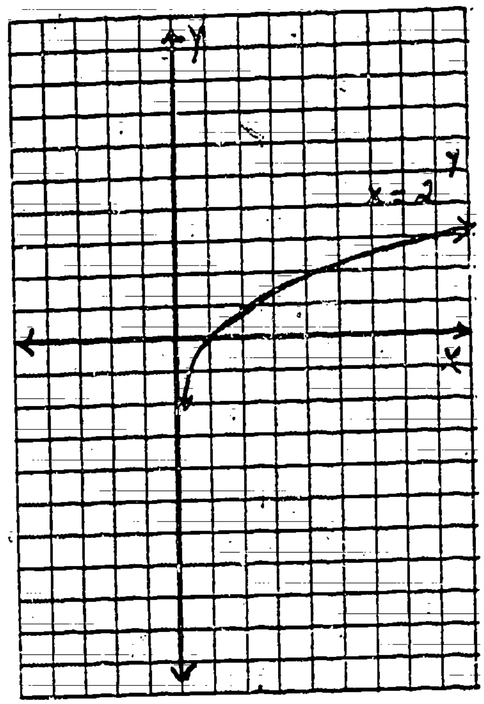


2.

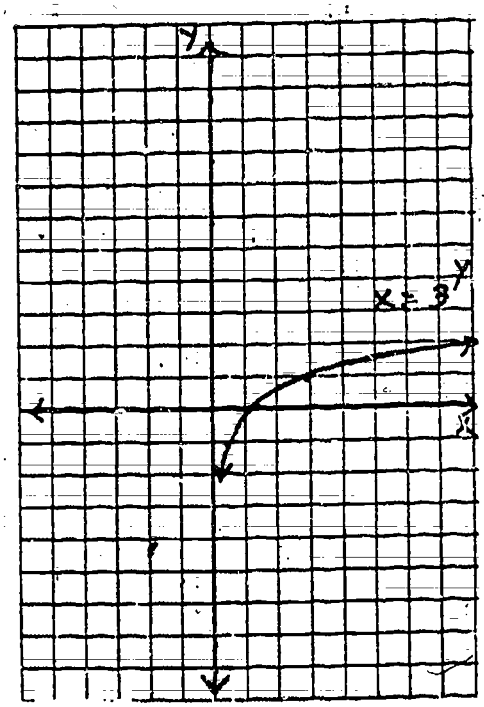


XI-2 (continued)

3.



4.



ANSWERS

XI-3

1. D = real numbers

   R = real numbers > 0

2. D = real numbers

   R = real numbers > 2

3. D = real numbers

   R = real numbers > 0

4. D = real numbers

   R = real numbers > -2

XI-4

1. $3^2 = 9$
2. $2^5 = 32$
3. $x^3 = x^3$
4. $10^4 = 10{,}000$

XI-5

1. $\log_8 64 = 2$
2. $\log_3 243 = 5$
3. $\log_x 64 = 6$
4. $\log_x 729 = 3$

XI-6

1. $\{4\}$
2. $\{\frac{1}{32}\}$
3. $\{3\}$
4. $\{-5\}$

XI-7

1. $\log\ 17 + \log\ 43$
2. $\log_3 2 - \log_5 3$
3. $\log_5 2 + \log_5 3$
4. $\log\ 7 + \log\ 3 - \log\ 11$

XI-8

1. $5 \log\ 3$
2. $2 \log_5 7$
3. $\frac{1}{2} \log_5 17$
4. $2 \log\ 3 - 2 \log\ 5$ or

   $2(\log\ 3 - \log\ 5)$

XI-9

1. c)
2. b)
3. d)
4. d)

XI-10

1. a) .8000

   b) .8976

   c) .9191

   d) .6990

   e) .3243

ANSWERS

XI-10 (continued)

2. a) .5391
   b) .4314
   c) .9652
   d) .6998
   e) .7782
3. a) .9047
   b) .6128
   c) .9996
   d) .4265
   e) .4771
4. a) .6767
   b) .5911
   c) .7490
   d) .9042
   e) .8451

XI-11

1. a) 2
   b) -2
   c) 0
   d) 1
   e) -5

XI-11 (continued)

2. a) -1
   b) 0
   c) 3
   d) 9
   e) -4
3. a) 5
   b) -3
   c) 1
   d) 0
   e) -8
4. a) 3
   b) 1
   c) 0
   d) -3
   e) -1

XI-12

1. a) 3
   b) .6291
2. a) 0
   b) .1430
3. a) 2
   b) .7853
4. a) -3
   b) .2967

ANSWERS

XI-13

1. 3.6964
2. 6.7482 - 10
3. 1.2788
4. [illegible]074 - [illegible]0

XI-14

1. 825
2. 0.0412
3. 228,000
4. 0.00753

XI-15

1. c)
2. a)
3. b)
4. c)

XI-16

1. 2.0762
2. 9.3213-10
3. 7.6606
4. 8.5894-10

XI-17

1. {9}
2. {24}
3. {-1, 3}
4. $\{\frac{2}{3}\}$

XI-18

1. 0.3867
2. 490.6
3. 2.8408
4. 0.009506

XI-19

1. 8.85 inches
2. $3738.55
3. $8144.47
4. 13.0 years

XI-20

1. 1.893
2. 1.086
3. 0.2432
4. 2.795

HIGHER ORDER ASSESSMENT TASKS

1. a) 50
   b) 100
   c) 200

2. a) 180
   b) 20
   c) 50

UNIT XII - TRIGONOMETRY

PURPOSE

This unit is designed to introduce stud   to two kinds of periodic functions, and to develop a working knowledge of the fundamental properties and methods of trigonometry.

OVERVIEW

This unit is an introduction to trigonometry with the definitions of the circular functions given in terms of coordinates of points on the unit circle and the definition of the trigonometric functions given in terms of coordinates of points on the terminal side of an angle in standard position. These two periodic functions are related by means of radian angle measure. Determining functions of angles and arc lengths is developed through use of coordinates of points on the unit circle and through use of tables of values of the trigonometric functions. The fundamental identities are developed and these identities are used to prove other identities and to derive the formulas necessary to solve triangles. The sketch of the graphs of the six circular functions and their inverses is developed. Solutions both general and particular are determined for trigonometric equations.

SUGGESTIONS TO THE TEACHER AND UNIT OUTLINE

Part I: Objectives 1-17

This unit covers t basic definitions c the trigonometric functions and how to use the unit c e or tables to find the values of the trigonometric functions for any angle

Time: 7-10 days

Part II: Objectives 18-20

This is a short unit on graphing the trigonometric functions.

Time: 6 days

Part III: Objectives 21-25

This unit covers the proofs of the basic trigonometry identities and some work with verifying identities.

Time: 8-10 days

PART IV: Objectives 26-28

This is a short study of the inverse trigonometric functions.

Time: 8 days

Part V: Objectives 29-34

This section deals with the solutions of triangles.

Time: 7-10 days

The total time spent on the trigonometry unit should be 7-9 weeks. It is suggested that the order of completion of this unit be Part I, Part II, and Part IV. Then do Part III and Part V in the order which best supports the students, best suits the remaining time and the courses to be taken by the students the following year.

The following should be noted:

- All basic identities should be derived; not simply stated.
- Students should understand the relationship between the circular function and the trigonometric functions.
- Curves should be sketched using only critical points, and the x-axis should be marked off in radian measure. Students should not be allowed to use degree measure on the x-axis.
- Students will need much practice in working with the trigonometric identities and solving trigonometric equations.

Computer Applications: BASIC BASIC, Coan, pp. 124-128 (Chapter 9: Trigonometry), 141-145; Algebra 2 and Trigonometry, Dolciani (1978), pp. 486, 511, 516; Algebra 2 and Trigonometry, Dolciani, (1980), pp. 509, 535, 541; Computer Programming in the BASIC Language, Golden, pp. 32 (#85, 86), 33 (#95), 56-57, 68 (#94-97), 97 (#72, 78), 98 (#79, 80, 88), 99 (#97-102, 104), 139 (#70, 77), 140 (#82, 83), 171 (#42-44); Algebra Two with Trigonometry, Payne, pp. 528-530; Modern Algebra and Trigonometry, Sorgenfrey, p. 511.

ENRICHMENT

Additional assessments are provided to assist instruction of polar form, graphing on the polar coordinate plane, and the use of de Moivre's Theorem.

VOCABULARY

wrapping function
trigonometric functions
circular functions
identity
trigonometric equations
radian measur
degree measure
sine
cosine
tangent
cosecant
secant
cotangent

period
amplitude
phase shift
Pythagorean Identities
inverse function
Arctan
Arccot
Arcsec
Csc^{-1}
Sin^{-1}
Cos^{-1}

Law of Sines
Law of Cosines

UNIT XII - TRIGONOMETRY

PERFORMANCE OBJECTIVES

1. Name the point in the unit circle, $x^2 + y^2 = 1$, located by the wrapping function, w.

2. Determine sin θ and cos θ, given the coordinator of a poir on the unit circle.

3. Determine the sin θ and cos θ for given values of θ.

4. Determine the sin θ and cos θ of an angle θ in stand position, given the coordinates of a point on its terminal side.

5. Given the quadrant of the terminal side of θ and either cos θ or sin θ, apply the basic identity $\cos^2\theta + \sin^2\theta = 1$ to determine the value of either the sin θ or the cos θ.

6. Define tan θ, cot θ, sec θ, and csc θ in terms of sin θ and/or cos θ.

7. Determine the tan θ, cot θ, sec θ, or csc θ, given sin θ and/or cos θ and/or the quadrant of the terminal side of θ.

8. Define the trigonometric functions in term of the sides and angles of a given right triangle.

9. Convert from radian measure to degree measure.

10. Convert from degree measure to radian measure.

11. Determine the values of the trigonometric functions of 45°, 30°, 60° and their multiples between 0° and 360°.

12. Given the measure of an angle in standard position, determine the measure of its reference angle.

13. Giv he measure of an angle to the neare t ten minutes, use the table to determine the given function value.

14. Given the measure of an angle to the nearest minute, use the table to determine the given function value to four significant digits.

15. Determine the measure of θ (to the near t 10 minutes) for the first quadrant angle with the given function value.

16. Det ine the measure of θ in degrees and minutes (to the nearest minute) for $0 \leq \theta \leq 360°$, with iven function value.

17. Use the periodic pr pert and table of values of the sine, cosine, and tangent functions to evaluate sin θ, cos θ, or tan θ, when θ > 360°.

18. Sketch the graph of each of the six circular functions over two fundamental periods of the curve.

19. Given the equation of a trigonometric function, determine:

   a) the amplitude

   b) the period

   c) the phase shift

   d) the vertical translation

20. Sketch the graph of the given function over one period.

21. Write a proof of the Pythagorean Identities.

22. Write a proof of the sine, cosine, and tangent of angle sums or differences.

23. Apply the sum and difference formulas in writing the proof of a reduction formula.

24. Apply the sum and difference formulas to determine the sine, cosine, or tangent of a given angle.

25. Write a proof of the sin 2θ, cos 2θ, and tan 2θ, given sine, cosine, tangent of the sum of 2 angles.

26. Write a proof of the half-angle formulas given the double-angle formulas.

27. Use the basic identities to simplify a complex trigonometric expression to a single function value.

28. Use the basic trigonometric identities to verify other identities.

29. Sketch the graph of the inverses of a given circular function and state the domain and range of the portion of the graph that is its principal-value function.

30. Determine the value of an expression involving inverse trigonometric functions.

31. Determine the solution(s) of trigonometric equations. State both the general and particular solution(s).

32. Use the trigonometric relationships in a right triangle to solve narrative problems.

33. Apply the Law of Cosines to solve a triangle.

34. Apply the Law of Sines to a triangle, when given two angles and one side of a triangle.

35. Solve a determined number of triangles, given two sides and an angle opposite one of them.

36. Determine the area of a triangle, given two sides and the included angle of the triangle.

37. Determine from the given data which of the laws to apply initially in solving a given triangle.

ENRICHMENT OBJECTIVES

1. Given the coordinates of a point in Cartesian form, determine a pair of coordinates of the point in polar form.

2. Given the coordinates of a point in polar form, determine the Cartesian coordinates of the point.

3. Given an equation in polar form, determine an equivalent equation in rectangular form.

4. Given an equation in rectangular form, determine an equivalent equation in polar form.

5. Sketch the graph of a given polar equation.

6. Express a given complex number (x + y i) in polar form.

7. Determine the product of quotient of complex numbers in polar form.

8. Use DeMoivre's Theorem to determine roots and powers of complex numbers.

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (197)
1	463-464	483	505-508	--	596 598	--	407-4[illegible]	--	--
2	466-469 371-374	487	510-514	452-455	599	--	421	430-434	427
3	466-469	491-494	514-518	452-455	--	--	408-409 411-414	430-434	428
4	371-374	487	514-518	--	599	442-447	421	427-430	427-428
5	--	489-490	510-514	463-465	596	--	409-410	--	431-433
6	421-422 377-378	506	530-534	463-465	599-600	488-489	423-424 426	434-435	435
7	379-382	506-508	[illegible]	[illegible]	601	442-446	427-429	--	436-438
8	383	512	536-5[illegible]	488	592-593	443	462-463	440-441	443
9	463-466	484-485	505-508	448-450	596-598	478-480	422-423 425	514-517	427-429
10	463-466	484-485	508	448-450	596-598	478-480	422-423 425	514-517	427-429

XII-6

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CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
11	382-385	--	514-518	452-455	601-603	447-450	423	439-443	436-437
12	395-398	499-500	521-523	452-455	603	442-447	453	452-453	436
13	386-394	494-496	518-520	491-493	622-624 626	454-458	451-455	443-452	439-441
14	386-394	495-497	518-520	491-493	624-626	454-458	456-458	443-452	439-441
15	386-394	496-497	518-520	491-493	625-626	454-458	454-455	443-452	439-441
16	386-394	496-497	521-523	491-493	625-626	454-458	457-458	443-452	439-441
17	395-398	493-494	521-523	--	625-626	454-458	--	443-452	429
18	470-477	501-502 508	524-529	456-462	606-610	482-487	415-417 424 428-429	520-527	436-437
19	470-477	502-505 508-511	524-534	456-462	619-622	482-487	416-421	520-527	--
20	470-476	505-505 508-[illegible]	524-534	456-462	619-622	482-487	418-421	520-527	436-437

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
21	422	489 519-520	545-5[illegible]	[illegible]-465	613-614	488 489	411 426-427	472-47[illegible]	432 460
22	430-431 433-434	525-528 532-534	551-558	469-471	635-638	491-492	433-436	48[illegible]-[illegible]85	[illegible]-467
23		528-530 538 535-537	551-558	469-471	639	--	436		--
24	435-436	529-530 535-537	551-558	469-471	638-639	493-494	435	483-484	469
25	436-439	537-538	565-568	472-474	640-641	497	437-438	483-487	470
26	436-439	538-539	565-568	472-474	642-643	499	437-438	487-488	470
27	426-427	521-522	568-570	--	--	494-497	432	--	462-464
28	426-427	522-524 537 540, 542	568-570	466-468	645-647	497	430-433 439-440	477-478	462-464
29	480-485	553-555	583-589	478-481	648-651	498-500	44[illegible]-4[illegible]	527-5[illegible]	--
30	484-485	555-558	583-589	478-481	649-652	500	442-443 454-455	530-535	--

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
31	485-487	558-561	589-597	475-477	653-656	501-503	459-462	535-539	474-476
32	389-391	515	537-540	495-500	659-661	452-454	465-466	451-452	444-445
33	444-447	544-546	571-574	505-507	667-669	468-470	471-473	494	449-451
34	447-450	547-549	574-578	502-504	662-663 666	466-468	466-468 470	497-498	446-448
35	447-450	548	574-578	508-510	663-666	468-470	468-470	494	449-451
36	447,456 457	546-547 549	574-578	--	--	466-468	--	506-507	446-448
37	450-456	--	571-579	502-510	--	--	--	501-506	--
ENRICHMENT	--	561-572	593-604	--	670-675	--	474-483	--	--

XII-9

PERFORMANCE OBJECTIVE XII-1

Name the point in the unit circle $x^2 + y^2 = 1$, located by the wrapping function w.

1. Name the point in the unit circle located by $w(\pi)$.
2. Name the point in the unit circle located by $w\left(\frac{3\pi}{2}\right)$.
3. Name the point in the unit circle located by w (5 [illegible]).
4. Name the point in the unit circle located by $w\left(\frac{7\pi}{2}\right)$.
5. Name the point in the unit circle located by $w\left(\frac{^-3\pi}{2}\right)$.

PERFORMANCE OBJECTIVE XII-2

Determine $\sin\theta$ and $\cos\theta$ given the coordinates of a point on the unit circle.

1. Given: $P(1, 0)$ as the coordinates of a point on the unit circle, determine the $\sin\theta$ and $\cos\theta$.
2. Given: $P(0, -1)$ as the coordinates of a point on the unit circle, determine the $\sin\theta$ and $\cos\theta$.
3. Given: $P(^-a, b)$ as the coordinates of a point on a unit circle, determine the $\sin\theta$ and $\cos\theta$.
4. Given: $P(^-a, ^-b)$ as coordinates of a point on a unit circle, determine the $\sin\theta$ and $\cos\theta$.

PERFORMANCE OBJECTIVE XII-3

Determine the sin θ and cos θ for given values of θ.

1. $\sin \pi = ?$

   $\cos \pi = ?$

2. $\sin\left(-\frac{\pi}{2}\right) = ?$

   $\cos\left(-\frac{\pi}{2}\right) = ?$

3. $\sin \frac{\pi}{4} = ?$

   $\cos \frac{\pi}{4} = ?$

4. $\sin \frac{5\pi}{2} = ?$

   $\cos \frac{5\pi}{2} = ?$

PERFORMANCE OBJECTIVE XII-4

Determine the sin θ and cos θ of an angle θ in standard position, given the coordinates of a point on its terminal side.

1. Determine the sin θ if (1, 2) are the coordinates of a point on the terminal side of θ in standard position.
2. Determine the cos θ if (-3, -4) are the coordinates of a point on the terminal side of θ in standard position.
3. Determine the cos(-θ) if (-2, 1) are the coordinates of a point on the terminal side of (-θ) in standard position.
4. Determine the sin(-θ) if (3, -4) are the coordinates of a point on the terminal side of θ in standard position.

PERFORMANCE OBJECTIVE XII-5

Given the quadrant of the terminal side of θ and either $\cos\theta$ or $\sin\theta$ apply the basic identify $\cos^2\theta + \sin^2\theta = 1$ to determine the value of either the $\sin\theta$ or the $\cos\theta$.

1. Use the identity $\cos^2\theta + \sin^2\theta = 1$ to determine the $\sin\theta$ if $\cos\theta = -\frac{1}{2}$ and θ terminates in the second quadrant.
2. Use the identity $\cos^2\theta + \sin^2\theta = 1$ to determine the $\cos\theta$ if $\sin\theta = -\frac{3}{5}$ and θ terminates in the third quadrant.
3. Use the identity $\cos^2\theta + \sin^2\theta = 1$ to determine the $\cos(-\theta)$ if $\sin(-\theta) = \frac{\sqrt{3}}{2}$ and θ terminates in the first quadrant.
4. Use the identity $\cos^2\theta + \sin^2\theta = 1$ to determine the $\sin\theta$ if the $\cos\theta = .7071$ and θ terminates in the fourth quadrant.

PERFORMANCE OBJECTIVE XII-6

Define the $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$ in terms of $\sin\theta$ and/or $\cos\theta$.

1. Define the $\tan\theta$ in terms of the $\sin\theta$ and/or $\cos\theta$.
2. Define the $\cot\theta$ in terms of the $\sin\theta$ and/or $\cos\theta$.
3. Define the $\sec\theta$ in terms of the $\sin\theta$ and/or $\cos\theta$.
4. Define the $\csc\theta$ in terms of $\sin\theta$ and/or $\cos\theta$.

PERFORMANCE OBJECTIVE XII-7

Determine the tan θ, cot θ, sec θ, or csc θ, given the sin θ and/or the cos θ and/or the quadrant of the terminal side of θ.

1. If $\sin\theta = \frac{3}{2}$ and $\cos\theta = \frac{1}{2}$, determine the value of the tan θ.

2. If $\sin\theta = \frac{a}{c}$ and θ is an acute angle, determine the value of the sec θ.

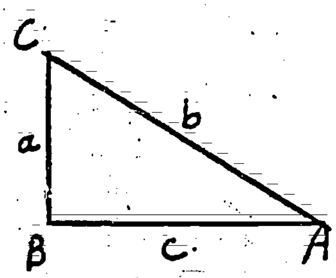
3. If $\cos\theta = t$ and θ is an acute angle, determine the value of cot θ.

4. If the $\cos\theta = \frac{5}{13}$ and $\pi < \theta < \frac{3\pi}{2}$, determine the value of csc θ.

PERFORMANCE OBJECTIVE XII-8

Define the trigonometric functions in terms of the sides and angles of a given right triangle.

1. Given right Δ ABC, right angle at B, determine the sin A, cos A, tan A, cot A, csc A, sec A.



sin A =

cos A =

tan A =

cot A =

csc A =

sec A =

PERFORMANCE OBJECTIVE XII-9

Convert from radian measure to degree measure.

1. Convert $\frac{5\pi}{6}$ to degree measure.

2. Convert $-\frac{9\pi}{4}$ to degree measure.

3. Convert -2 to degree measure.

4. Convert $\frac{\pi}{12}$ to degree measure.

PERFORMANCE OBJECTIVE XII-10

Convert from degree measure to radian measure.

1. Convert 140° to radian measure. Express your results in terms of π.

2. Convert -30° to radian measure. Express your results in terms of π.

3. Convert -540° to radian measure. Express your results in terms of π.

4. Convert 330° to radian measure. Express your results in terms of π.

PERFORMANCE OBJECTIVE XII-11

Determine the values of the trigonometric functions of 45°, 30°, 60°, and their multiples between 0 and 360°.

1. Using isosceles right triangle ACB with right angle at C and angle A in standard position, determine $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.

2. Using $30^\circ - 60^\circ$ right triangle with the 30° angle in standard position, determine the $\sin 30^\circ$, $\cos 30^\circ$, $\tan 30^\circ$.

3. Using $30^\circ - 60^\circ$ right triangle with the 60° angle in standard position, determine the $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$.

4. Using $30^\circ - 60^\circ$ right triangle, determine the $\sin 240^\circ$, $\cos 240^\circ$, $\tan 240^\circ$.

PERFORMANCE OBJECTIVE XII-12

Given the measure of an angle in standard position, determine the measure of its reference angle.

1. Given an angle 320° in standard position, determine the measure of its reference angle.

2. Given an angle (-120°) in standard position, determine the measure of its reference angle.

3. Given an angle 2^R in standard position, determine the measure of its reference angle (use $\pi = 3.14$).

4. Given an angle $\frac{7\pi^R}{6}$ in standard position, determine the measure of its reference angle.

PERFORMANCE OBJECTIVE XII-13

Given the measure of an angle to the nearest ten minutes, use the table to determine the given function value.

1. cos 197° 50'

2. tan (-261° 30')

3. sin 161° 40'

4. csc 300° 20'

PERFORMANCE OBJECTIVE XII-14

Given the measure of an angle to the nearest minute, use the table to determine the given function value to four significant digits.

1. sin 22° 54'

2. cos (-57° 42')

3. tan 145° 23'

4. cot 82° 36'

PERFORMANCE OBJECTIVE XII-15

Determine the measure of θ (to the nearest ten minutes) for the first quadrant angle with the given function value.

1. $\cos \theta = .6604$

2. $\cot \theta = 2.605$

3. $\csc \theta = 4.620$

4. $\tan \theta = 1.228$

PERFORMANCE OBJECTIVE XII-16

Determine the measure of θ in degrees and minutes (to the nearest minute) for $0 \leq \theta \leq 360^\circ$, with the given function value.

1. $\cos \theta \doteq .7848$, $\sin \theta > 0$

2. $\sin \theta \doteq .7742$, $\cos \theta > 0$

3. $\tan \theta \doteq 6.500$, $\sin \theta < 0$

4. $\sin \theta \doteq -0.5773$, $\cos \theta > 0$

PERFORMANCE OBJECTIVE XII-17

Use the periodic properties and table of values of the sine, cosine, and tangent functions to determine $\sin\theta$, $\cos\theta$, or $\tan\theta$, when $\theta > 2\pi$ or $\theta > 360^\circ$.

1. $\cos 1110^\circ =$

2. $\sin\frac{9\pi}{4} =$

3. $\sin(-660^\circ) =$

4. $\cos\left(\frac{-13\pi}{4}\right) =$

5. $\tan 420^\circ =$

6. $\tan\left(\frac{-11\pi}{4}\right) =$

PERFORMANCE OBJECTIVE XII-18

Sketch the graph of each of the six circular functions over two fundamental periods of the curve.

1. Sketch $\{(x, y): y = \sin x, -2\pi \leq x \leq 2\pi\}$

2. Sketch $\{(x, y): y = \tan x, -2\pi \leq x \leq 2\pi\}$

3. Sketch $\{(x, y): y = \cos x, -\pi \leq x \leq \pi\}$

4. Sketch $(x, y): y = \cot x, -\pi \leq x \leq \pi\}$

5. Sketch $\{(x, y): y = \sec x, -2\pi \leq x \leq 2\pi\}$

6. Sketch $\{(x, y): y = \csc x, -2\pi \leq x \leq 2\pi\}$

PERFORMANCE OBJECTIVE XII-19

Given the equation of a trigonometric function, determine:

a) the amplitude

b) the period

c) the phase shift

d) the vertical translation

1. $f(x) = 2 \sin \frac{1}{2} (x - \pi)$

2. $f(x) = 3 \cos (2x + \frac{\pi}{2})$

3. $f(x) = 2 \tan (x + \frac{\pi}{4}) + 2$

4. $f(x) = \csc 3x$

5. $f(x) = \frac{1}{2} \sec(x - \frac{\pi}{2}) - 3$

PERFORMANCE OBJECTIVE XII-20

Sketch the graph of the given function over one period.

1. Sketch the graph of $f(x) = 2 \sin \frac{1}{2} (x + \frac{\pi}{2})$.

2. Sketch the graph of $f(x) = -2 \tan (x + \frac{\pi}{4})$.

3. Sketch the graph of $f(x) = \csc 2x$.

4. Sketch the graph of $f(x) = 3 \cos (x - \frac{\pi}{3}) + 2$.

5. Sketch the graph of $f(x) = 2 \sec (2x + \frac{\pi}{2}) + 1$.

PERFORMANCE OBJECTIVE XII-21

Write a proof of the Pythagorean identities.

1. Write a proof of $\sin^2 \theta + \cos^2 \theta = 1$.

2. Write a proof of $1 + \cot^2 \theta = \csc^2 \theta$.

3. Write a proof of $\tan^2 \theta + 1 = \sec^2 \theta$.

PERFORMANCE OBJECTIVE XII-22

Write a proof of the sine, cosine, and tangent of angle sums or differences.

1. Write a proof of the $\cos(\theta_1 - \theta_2) = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2$.

2. Write a proof of the $\cos(\theta_1 + \theta_2) = \cos \theta_1 \cos_2 - \sin \theta_1 \sin\theta_2$.

3. Write a proof of $\sin(\theta_1 + \theta_2) = \sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2$.

4. Write a proof of $\sin(\theta_1 - \theta_2) = \sin \theta_1 \cos \theta_2 - \cos \theta_1 \sin \theta_2$.

5. Write a proof of $\tan(\theta_1 + \theta_2) = \dfrac{\tan \theta_1 + \tan \theta_2}{1 - \tan \theta_1 \tan \theta_2}$.

6. Write a proof of $\tan(\theta_1 - \theta_2) = \dfrac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$.

PERFORMANCE OBJECTIVE XII-23

> Apply the sum and difference formulas in writing the proof of a reduction formula.

1. Apply the formula for $\cos(\theta_1 \pm \theta_2)$ to prove $\cos(\frac{\pi}{2} \pm \theta) = \pm \sin\theta$.

2. Apply the formula for $\cos(\theta_1 \pm \theta_2)$ to prove $\cos(\pi \pm \theta) = -\cos\theta$.

3. Apply the formula for $\cos(\theta_1 \pm \theta)$ to prove $\cos(\frac{3\pi}{2} \pm \theta) = \pm \sin\theta$.

4. Apply the formula for $\sin(\theta_1 \pm \theta_2)$ to prove $\sin(\frac{\pi}{2} \pm \theta) = \cos\theta$.

5. Apply the formula for $\sin(\theta_1 \pm \theta_2)$ to prove $\sin(\pi \pm \theta) = \pm \sin\theta$.

6. Apply the formula for $\sin(\theta_1 \pm \theta_2)$ to prove $\sin(\frac{3\pi}{2} \pm \theta) = -\cos\theta$.

7. Apply the formula for $\tan(\theta_1 \pm \theta_2)$ to prove $\tan(\pi \pm \theta) = \pm \tan\theta$.

PERFORMANCE OBJECTIVE XII-24

> Apply the sum and difference formulas to determine the sine, cosine, or tangent of a given angle.

1. Apply the formula for $\cos(\theta_1 + \theta_2)$ to determine $\cos\frac{7\pi}{12}$.

2. Apply the formula for $\sin(\theta_1 - \theta_2)$ to determine $\sin\frac{\pi}{12}$.

3. Apply the formula for $\sin(\theta_1 + \theta_2)$ to determine $\sin(\frac{13\pi}{4})$.

4. Apply the formula for $\tan(\theta_1 + \theta_2)$ to determine $\tan(75^\circ)$.

PERFORMANCE OBJECTIVE XII-25

Write a proof of the $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

1. Apply the formula for $\sin(\theta + \theta_1)$ to prove that $\sin 2\theta = 2 \sin\theta \cos\theta$.

2. Apply the formula for $\cos(\theta + \theta_1)$ to prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$. Using $\cos^2\theta + \sin^2\theta = 1$, determine that $\cos 2\theta = 2\cos^2\theta - 1$ and $\cos 2\theta = 1 - 2\sin^2\theta$.

3. Apply the formula for $\tan(\theta + \theta_1)$ to prove that $\tan 2\theta = \frac{2 \tan\theta}{1 - \tan^2\theta}$.

PERFORMANCE OBJECTIVE XII-26

Write a proof of the half angle formulas given the double angle formulas.

1. Write a proof of $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$ given that $\cos 2\theta = 2\cos^2\theta - 1$.

2. Write a proof of $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$ given that $\cos 2\theta = 1 - 2\sin^2\theta$.

3. Write proof of $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$ given $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$.

PERFORMANCE OBJECTIVE XII-27

Use the basic identities to simplify a complex trigonometric expression to a single function value.

1. Using the basic identities, express $\frac{2 \sin \theta}{\sin \theta \cot \theta + \cos \theta}$ in terms of the $\tan \theta$.

2. Using the basic identities, express $\cos \theta \sec \theta + \frac{\cos \theta}{\sin \theta \tan \theta}$ in terms of $\csc \theta$.

3. Using the basic identities, express $\frac{\cos \theta}{\sec \theta - \tan \theta}$ in terms of $\sin \theta$.

4. Using the basic identities, express $\frac{\sec \theta}{\cot \theta + \tan \theta}$ in terms of $\sin \theta$.

PERFORMANCE OBJECTIVE XII-28

Use the basic trigonometric identities to verify other identities.

1. Use the basic identities to verify:

$$\frac{1 + \tan^2 x}{\tan^2 x} = \csc^2 x$$

2. Use the basic identities to verify:

$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

3. Use the basic identities to verify:

$$\frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$$

4. Use the basic identities to verify:

$$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2 \sin^2 \theta - 1$$

PERFORMANCE OBJECTIVE XII-29

Sketch the graph of the inverse of a given circular function and state the domain and range of the portion of the graph that is the principal-value function.

1. Sketch the graph of $y = \mathrm{Sin}^{-1} x$ and state the domain and range of the principal-value function for sine.
2. Sketch the graph of $y = \mathrm{Cos}^{-1} x$ and state the domain and range of the principal-value function for cosine.
3. Sketch the graph of $y = \mathrm{Arc} \tan x$ and state the domain and range of the principal-value function for tangent.
4. Sketch the graph of $y = \mathrm{Arc} \cot x$ and state the domain and range of the principal-value function for cotangent.
5. Sketch the graph of $y = \mathrm{Arc} \sec x$ and state the domain and range of the principal-value function for secant.
6. Sketch the graph of $y = \mathrm{Csc}^{-1} x$ and state the domain and range of the principal-value function for cosecant.

PERFORMANCE OBJECTIVE XII-30

Determine the value of an expression involving inverse trigonometric functions.

1. Determine: $\text{Tan}^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

2. Determine: Arc sin 0.3311.

3. Determine: $\sin\left[\text{Cos}^{-1}\left(-\frac{4}{5}\right)\right]$.

4. Determine: $\text{Sec}^{-1}(\cos 0^\circ)$.

5. Determine: $\text{Sin}\left(\text{Arc tan } \frac{1}{2} + \text{Arc tan } \frac{1}{3}\right)$.

PERFORMANCE OBJECTIVE XII-31

Determine the solution(s) of trigonometric equations.

State both the general and particular solution(s).

1. Determine the general solution of $2 \sin \theta + 3 = 4$.

2. Determine the particular solution for $\tan^2 x + \tan x = 0$ in the interval $0 \le \theta \le 2\pi$.

3. Determine the particular solution for $2 \sin^2 \theta + 3 \sin \theta - 2 = 0$ in the interval $0 \le \theta \le 2\pi$.

4. Determine both the general and particular solution for $(\cos 2\theta)(3 - 4 \sin^2 \theta = 0$. Determine the particular solution in the interval $0 \le \theta \le 360^\circ$.

HIGHER ORDER ASSESSMENT

Determine the particular solution for $\sin 3\theta + \sin \theta = \cos 3\theta + \cos \theta$ in the interval $0 \le \theta \le 90^\circ$.

PERFORMANCE OBJECTIVE XII-32

Use the trigonometric relationships in a right triangle to solve narrative problems.

1. Find the length of the altitude of an isosceles triangle whose base has length 20 inches and each of whose base angles has measure 54°.

2. Wesville is 200 km N 40° E of Blue Note, and due north of Rosas. Rosas is due east of Blue Note. Determine the distance from Rosas to Wesville.

3. A guy wire holding an antenna is 70 feet long and is attached to the antenna at a distance of 50 feet from the ground. Find the measure of the angle formed by the wire and the ground.

4. The angles of elevation to the top and bottom of a vertical flagpole on top of a building from a point 140 feet from the base of the building measure 59° and 57° 50' respectively. How tall is the flagpole?

5. Determine the measure of the interior angles of a rhombus if the area is 68.7 cm^2 and the length of one side is 12 cm.

6. Find the area of a parallelogram with sides of 10 inches and 17 inches if one angle has measure 65° 30'.

PERFORMANCE OBJECTIVE XII-33

Apply the Law of Cosines to solve a triangle.

1. In triangle ABC, $a = 6$, $b = 8$ and $m\angle C = 60^\circ$. Use the Law of Cosines to determine c to the nearest integer.

2. In triangle ABC, $b = 10$, $c = 6$ and $m\angle A = 150^\circ$. Use the Law of Cosines to determine a to the nearest integer.

3. In triangle ABC, $a = 6$, $b = 9$ and $c = 14$. Use the Law of Cosines to determine the measure of the largest angle of the triangle.

4. The sides of a rhombus measure 8 inches each. One of the angles of this figure is 100°. Find the longer diagonal, correct to the nearest tenth of an inch.

PERFORMANCE OBJECTIVE XII-34

Apply the Law of Sines to solve a triangle, when given two angles and one side of the triangle.

1. Use the Law of Sines to find the length of side b in triangle ABC if $a = 640$, $m\angle A = 70^\circ$, and $m\angle B = 52^\circ$.

2. Use the Law of Sines to find the length of side a in triangle ABC if $b = 600$, $m\angle B = 11^\circ$, and $m\angle C = 75^\circ$.

3. Use the Law of Sines to find the length of side a in triangle ABC if $c = 0.8$, $m\angle A = 25^\circ 30'$ and $m\angle B = 70^\circ 50'$.

4. Use the Law of Sines to find all the sides and angles in triangle ABC if $b = 15$, $m\angle A = 45^\circ$ and $m\angle C = 36^\circ 10'$.

PERFORMANCE OBJECTIVE XII - 35

Solve a determined number of triangles given two sides and an angle opposite one of them.

1. Determine how many triangles exist and, if any do, solve the triangle(s) if $a = 14$, $b = 18$, and $m\angle A = 35^{\circ}$.
2. Determine how many triangles exist and, if any do, solve the triangle(s) if $b = 136$, $c = 150$ and $m\angle B = 110^{\circ}$.
3. Determine how many triangles exist and, if any do, solve the triangle(s) if $b = 14$, $c = 7.8$, and $m\angle C = 31^{\circ}\ 20'$.
4. Determine how many triangles exist and, if any do, solve the triangle(s) if $a = 12$, $b = 6$, and $m\angle B = 32^{\circ}$.

PERFORMANCE OBJECTIVE XII-36

Determine the area of a triangle, given two sides and the included angle of the triangle.

1. Two sides of a triangle are 6 feet and 10 feet and the included angle is 150°. Determine the area of the triangle.
2. If two adjacent sides of a triangle are 5 and 8 and the included angle is 30°, determine the area of the triangle.
3. Determine the area of a triangular plot of ground if two of its adjacent sides are 19 rods and 14 rods and the included angle is 121°.
4. If the area of $\Delta ABC = 150$, $a = 100$ and $b = 25$, determine the measure of angle C to the nearest degree.
5. Determine the area of a parallelogram with diagonals of lengths 40 cm and 60 cm if the diagonals intersect at an angle of 54°.

PERFORMANCE OBJECTIVE XII-37

Determine from the given data which of the laws to apply initially in solving a given triangle.

1. Which law would be used first in solving triangle ABC if $a = 10$, $b = 7$, and $c = 14$?
2. Which law would be used first in solving triangle ABC if $a = 12$, $b = 10$, and $m\angle B = 100^\circ$?
3. Which law would be used first in solving triangle ABC if $a = 15$, $m\angle B = 47^\circ$, and $m\angle C = 70^\circ$?
4. Which law would be used first in solving triangle ABC if $a = 10$, $b = 12$, and $m\angle C = 70^\circ$?

ENRICHMENT 1

> Given the coordinates of a point in Cartesian form, determine a pair of coordinates of the point in polar form.

1. Given $(2, 2)$, determine a pair of coordinates of the point in polar form with $0^\circ \leq \theta \leq 90^\circ$.
2. Given $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$, determine a pair of coordinates of the point in polar form with $0^\circ \leq \theta \leq 180^\circ$.

ENRICHMENT 2

> Given the coordinates of a point in polar form, determine the Cartesian coordinates of the point.

1. Determine the Cartesian coordinates of $(5, 60^\circ)$.
2. Determine the Cartesian coordinates of $(-3\sqrt{2}, 135^\circ)$.

ENRICHMENT 3

Given an equation in polar form, determine an equivalent equation in rectangular form.

1. Determine the equation in rectangular form equivalent to $r = 4 \sin \theta$.
2. Determine the equation in rectangular form equivalent to $r = 2 \tan \theta$.

ENRICHMENT 4

Given an equation in rectangular form, determine an equivalent equation in polar form.

1. Determine the polar equation equivalent to $3x + y = 2$.
2. Determine the polar equation equivalent to $x^2 + y^2 + 2x = d$.

ENRICHMENT 5

Sketch the graph of a given polar equation.

1. Sketch the graph of $r = 2 \cos \theta$.

2. Sketch the graph of $r = \sin 2\theta$.

ENRICHMENT 6

Express a given complex number $(x + y\,i)$ in polar form.

1. Express $3 + 2i$ in polar form.

2. Express $\sqrt{3} - i$ in polar form.

ENRICHMENT 7

Determine the product of quotient of complex numbers in polar form.

1. Determine the product of 2 (cos 225°) and 3 (cos 75° + i sin 75°).

2. Determine the quotient of 4 (cos 125° + i sin 125°) and 5 (cos 95° + i sin 95°).

ENRICHMENT 8

Use DeMoivre's Theorem to determine roots and powers of complex numbers.

1. Use DeMoivre's Theorem to simplify $[2\ (\cos 30° + i \sin 30°)]^5$.

2. Use DeMoivre's Theorem to determine the three cube roots of 27 (cos 90° + i sin 90°).

3. Determine the solution set of $x^4 = 16$ over the set of complex numbers.

ANSWERS

XII-1

1. $(-1, 0)$
2. $(0, -1)$
3. $(-1, 0)$
4. $(0, -1)$
5. $(0, 1)$

XII-2

1. $\sin\theta = 0$ and $\cos\theta = 1$
2. $\sin\theta = -1$ and $\cos\theta = 0$
3. $\sin\theta = b$ and $\cos\theta = -a$
4. $\sin\theta = -b$ and $\cos\theta = -a$

XII-3

1. $\sin\pi = 0$

   $\cos\pi = 1$
2. $\sin(-\frac{\pi}{2}) = -1$

   $\cos(-\frac{\pi}{2}) = 0$
3. $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

   $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
4. $\sin\frac{5\pi}{2} = 1$

   $\cos\frac{5\pi}{2} = 0$

XII-4

1. $\frac{2\sqrt{5}}{5}$
2. $\frac{-3}{5}$
3. $\frac{-2\sqrt{5}}{5}$
4. $\frac{4}{5}$

XII-5

1. $\frac{\sqrt{3}}{2}$
2. $\frac{-4}{5}$
3. $\frac{1}{2}$
4. $-.7071$

XII-6

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\cot\theta = \frac{\cos\theta}{\sin\theta}$
3. $\sec\theta = \frac{1}{\cos\theta}$
4. $\csc\theta = \frac{1}{\sin\theta}$

XII-7

1. $\sqrt{3}$
2. $\frac{c}{\sqrt{c^2 - a^2}}$
3. $\frac{t}{\sqrt{1 - t^2}}$
4. $\frac{-13}{12}$

XII-8

1. $\sin A = \frac{a}{b}$
2. $\cos A = \frac{c}{b}$
3. $\tan A = \frac{a}{c}$
4. $\cot A = \frac{c}{a}$
5. $\csc A = \frac{b}{a}$
6. $\sec A = \frac{b}{c}$

ANSWERS

XII-9

1. $150°$

2. $-405°$

3. $\frac{-360°}{\pi}$

4. $15°$

XII-10

1. $\frac{7\pi}{9}$

2. $\frac{-\pi}{6}$

3. -3π

4. $\frac{11\pi}{6}$

XII-11

1. $\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45° = 1$

2. $\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

$\tan 30° = \frac{\sqrt{3}}{3}$

3. $\cos 60° = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\tan 60° = \sqrt{3}$

XII-11 (continued)

4. $\cos 240° = -\frac{1}{2}$

$\sin 240° = -\frac{\sqrt{3}}{2}$

$\tan 240° = \sqrt{3}$

XII-12

1. $40°$

2. $60°$

3. 1.14^R

4. $\frac{\pi}{6}$

XII-13

1. $= .9520$

2. $= 6.691$

3. $.3145$

4. $= 1.159$

ANSWERS

XII-14

1. .3891
2. .5343
3. -.6903
4. .1299

XII-15

1. 48° 40'
2. 21°
3. 12° 30'
4. 50° 50'

XII-16

1. 38° 18'
2. 50° 44'
3. 261° 15'
4. 324° 44'

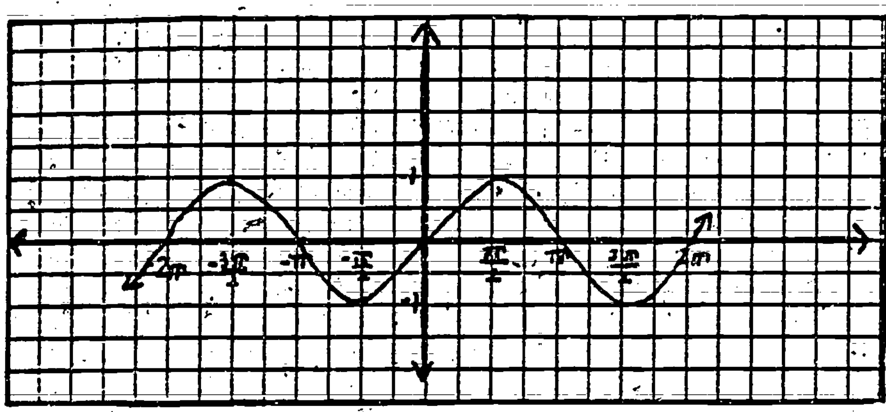
XII-17

1. .866
2. .7071
3. .866
4. -.7071
5. $\sqrt{3}$
6. 1

ANSWERS

XII-18

1.



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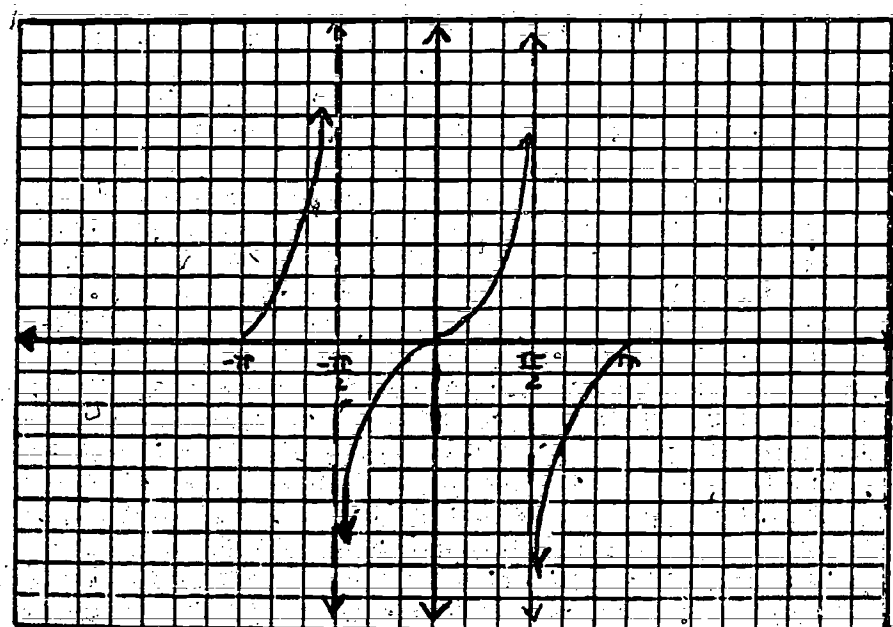

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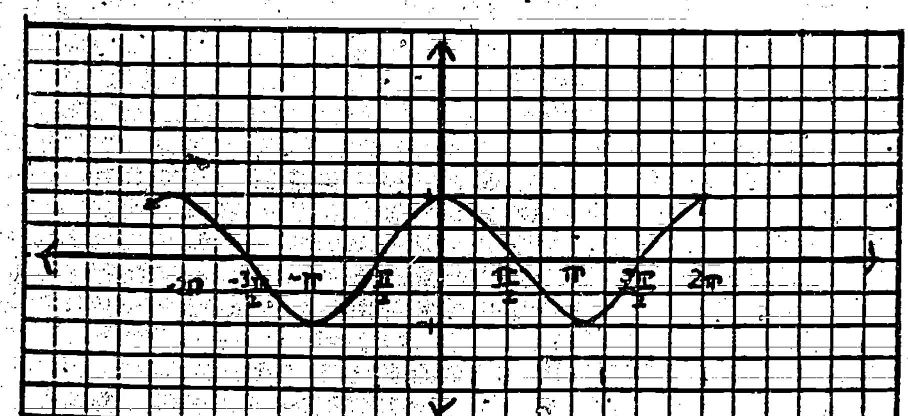
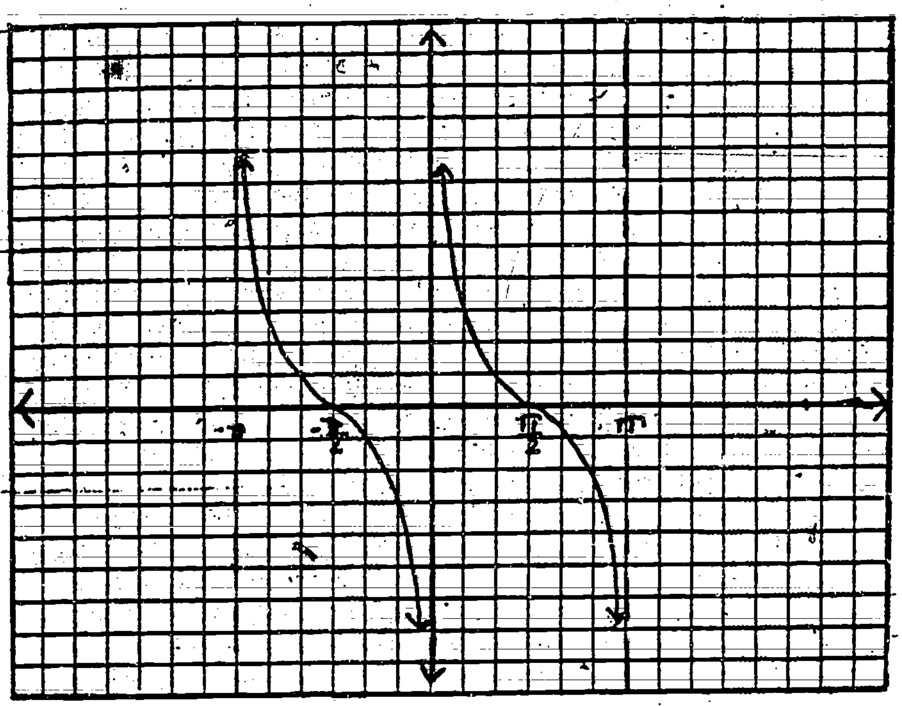


ANSWERS

XII-18 (continued)

4.



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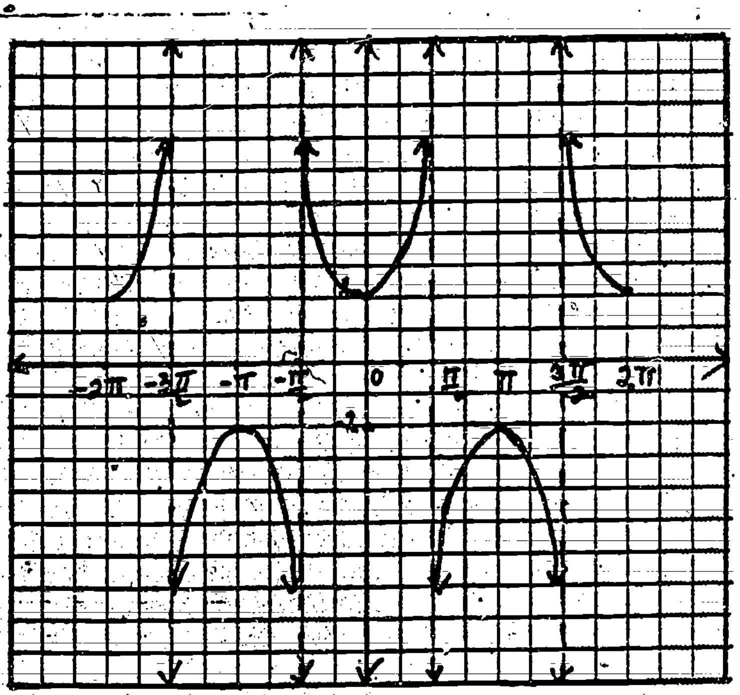
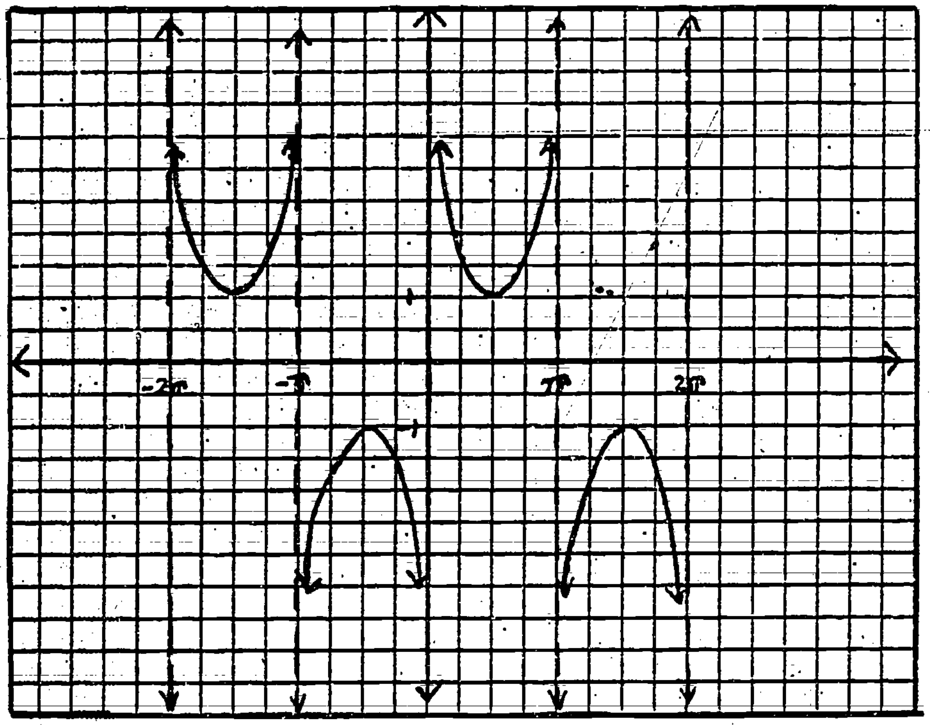


315

ANSWERS

XII-18 (continued)

6.



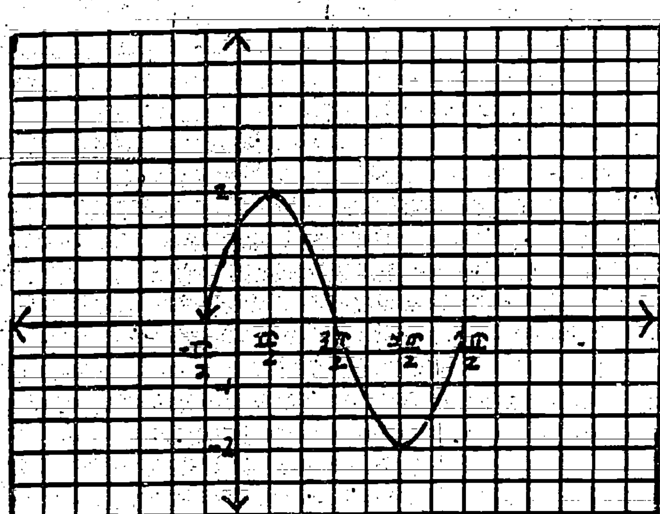
XII-19

1. a. 2
   b. 4π
   c. π to the right
   d. none
2. a. 3
   b. π
   c. $\frac{\pi}{4}$ to the left
   d. none
3. a. none
   b. π
   c. $\frac{\pi}{4}$ to the left
   d. up 2
4. a. None
   b. $\frac{2\pi}{3}$
   c. None
   d. None
5. a. None
   b. 2π
   c. $\frac{\pi}{2}$ to the right
   d. Down 3

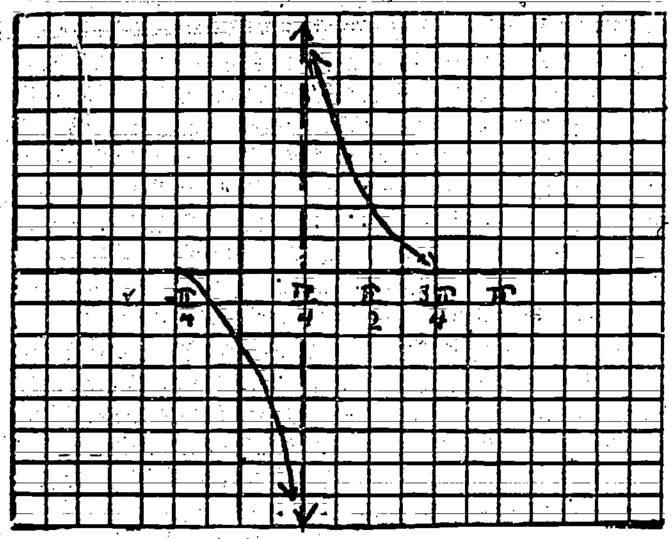
ANSWERS

XII-20

1.



2.



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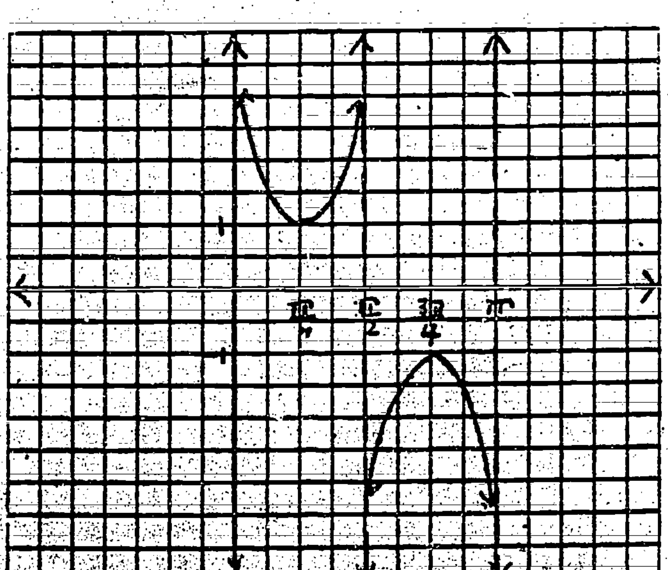
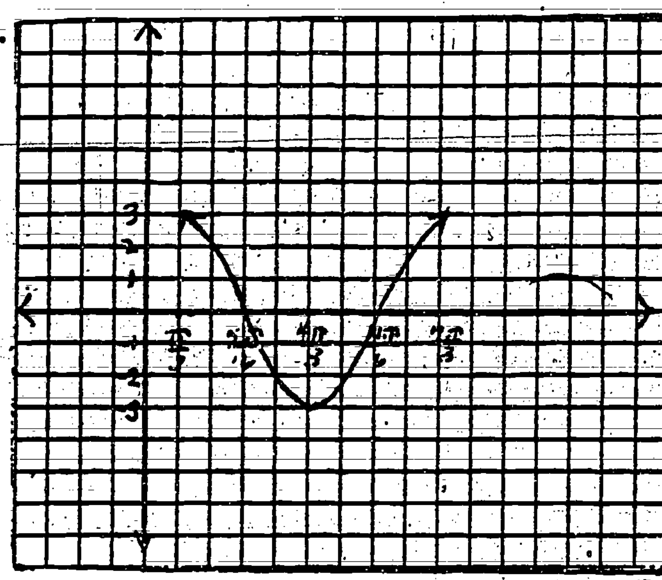
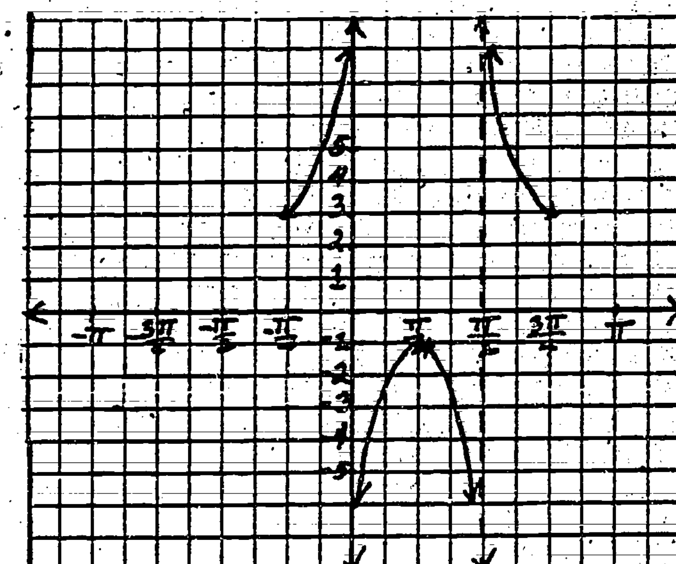


4.



5.



ANSWERS

XII-21

1. $x^2 + y^2 = R^2$

$\frac{x^2}{R^2} + \frac{y^2}{R^2} = 1$

$(\frac{x}{R})^2 + (\frac{y}{R})^2 = 1$

$\cos^2 \theta + \sin^2 \theta = 1$

2. $\sin^2 \theta + \cos^2 \theta = 1$

$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$

$1 + \cot^2 \theta = \csc^2 \theta$

3. $\sin^2 \theta + \cos^2 \theta = 1$

$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$

$\cos^2 \theta \neq 0$

$\tan^2 \theta + 1 = \sec^2 \theta$

XII-22

No answers are given. These can be found in any trigonometry book.

XII-23

1. $\cos(\frac{\pi}{2} \pm \theta) = \cos \frac{\pi}{2} \cos \theta \mp \sin \frac{\pi}{2} \sin \theta$

$= (0) \cos \theta \mp (1) \sin \theta$

$= \mp \sin \theta$

2. $\cos(\pi \pm \theta) = \cos \pi \cos \theta \mp \sin \pi \sin \theta$

$= (-1) \cos \theta \mp (0) (\sin \theta)$

$= - \cos \theta$

3. $\cos(\frac{3\pi}{2} \pm \theta) = \cos \frac{3\pi}{2} \cos \theta \mp \sin \frac{3\pi}{2} \sin \theta$

$= (0) \cos \theta \mp (-1) \sin \theta$

$\cos(\frac{3\pi}{2} \pm \theta) = \pm \sin \theta$

4. $\sin(\frac{\pi}{2} \pm \theta) = \sin \frac{\pi}{2} \cos \theta \pm \cos \frac{\pi}{2} \sin \theta$

$= (1) (\cos \theta) \pm (0) \sin \theta$

$\sin(\frac{\pi}{2} \pm \theta) = \cos \theta$

5. $\sin(\pi \pm \theta) = \sin \pi \cos \theta \pm \cos \pi \sin \theta$

$= (0) \cos \theta \pm (-1) \sin \theta$

$\sin(\pi \pm \theta) = \mp \sin \theta$

6. $\sin(\frac{3\pi}{2} \pm \theta) = \sin \frac{3\pi}{2} \cos \theta \quad \cos \frac{3\pi}{2} \sin \theta$

$= (-1) \cos \theta \pm 0 \sin \theta$

$\sin(\frac{3\pi}{2} \pm \theta) = -\cos \theta$

7. $\tan(\pi \pm \theta) = \frac{\tan \pi \pm \tan \theta}{1 \mp \tan \pi \tan \theta}$

$= \frac{\pm \tan \theta}{1}$

$\tan(\pi \pm \theta) = \pm \tan \theta$

ANSWERS

XII-24

1. $\cos \frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4}$

2. $\sin \frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$

3. $\sin (\frac{13\pi}{4}) = -\frac{\sqrt{2}}{2}$

4. $\tan 75° = \frac{\sqrt{3}+3}{3-\sqrt{3}}$

XII-25

1. $\sin (\theta + \theta_1) = \sin \theta \cos \theta_1 + \cos \theta \sin \theta_1$

   Replace θ_1 with θ

   $\therefore \sin (\theta + \theta) = \sin \theta \cos \theta + \cos \theta \sin \theta$

   $\sin 2\theta = 2 \sin \theta \cos \theta$

2. $\cos (\theta + \theta = \cos \theta \cos \theta_1 - \sin \theta \sin \theta_1$

   Replace θ_1 with θ

   $\cos (\theta + \theta) = \cos \theta \cos \theta - \sin \theta \sin \theta$

   a) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

   $\cos 2\theta = 1 - \sin^2 \theta - \sin^2 \theta$

   b) $\cos 2\theta = 1 - 2 \sin^2 \theta$

   $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

   $\cos \theta = \cos^2 \theta - (1 - \cos^2 \theta)$

   $= \cos^2 \theta - 1 + \cos^2 \theta$

   c) $\cos 2\theta = 2 \cos^2 \theta - 1$

3. $\tan (\theta + \theta_1) = \frac{\tan \theta + \tan \theta_1}{1 - \tan \theta \tan \theta_1}$

   Replace θ_1 with θ

   $\tan (\theta + \theta) = \frac{\tan \theta + \tan \theta}{1 - \tan \theta \tan \theta}$

   $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

XII-26

1. $\cos 2\theta = 2 \cos^2 \theta - 1$

   Replace θ with $\frac{\theta}{2}$

   $\cos 2 (\frac{\theta}{2}) = 2 \cos^2 (\frac{\theta}{2}) - 1$

   $\frac{\cos \theta + 1}{2} = \cos^2 (\frac{\theta}{2})$

   $\cos (\frac{\theta}{2}) = \pm \sqrt{\frac{\cos \theta + 1}{2}}$

2. $\cos 2\theta = 1 - 2 \sin^2 \theta$

   Replace θ with $(\frac{\theta}{2})$

   $\cos 2 (\frac{\theta}{2}) = 1 - 2 \sin^2 (\frac{\theta}{2})$

   $\sin^2 (\frac{\theta}{2}) = \frac{1 - \cos \theta}{2}$

   $\sin (\frac{\theta}{2}) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$

3. $\tan (\frac{\theta}{2}) = \frac{\sin (\frac{\theta}{2})}{\cos (\frac{\theta}{2})}$

   $= \frac{\pm \sqrt{\frac{1 - \cos \theta}{2}}}{\pm \sqrt{\frac{1 + \cos \theta}{2}}}$

   $= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$

XII-27

1. $\tan \theta$
2. $\csc^2 \theta$
3. $1 + \sin \theta$
4. $\sin \theta$

ANSWERS

XII-28 (Other methods are acceptable.)

1. $\frac{1 + \tan^2 x}{\tan^2 x} =$

$\frac{\sec^2 x}{\tan^2 x} =$

$\frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} =$

$\frac{1}{\sin^2 x} = \csc^2 x$

$\therefore \frac{1 + \tan^2 x}{\tan^2 x} = \csc^2 x$

2. $\frac{2 \tan x}{1 + \tan^2 x} =$

$\frac{2 \tan x}{\sec^2 x} =$

$2 \left(\frac{\sin x}{\cos x}\right) \cdot \cos^2 x =$

$2 \sin x \cos x = \sin 2x$

$\therefore \sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$

3. $\frac{\cos 2x}{\cos x - \sin x} =$

$\frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} =$

$\frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x - \sin x)} =$

$\cos x + \sin x$

$\therefore \frac{\cos 2x}{\cos x - \sin} = \cos x + \sin x$

4. $\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} =$

$\frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} =$

$\frac{\frac{\sin^2 \theta - \cos^2 \theta}{\cos \theta \sin \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}} =$

$\sin^2 \theta - \cos^2 \theta =$

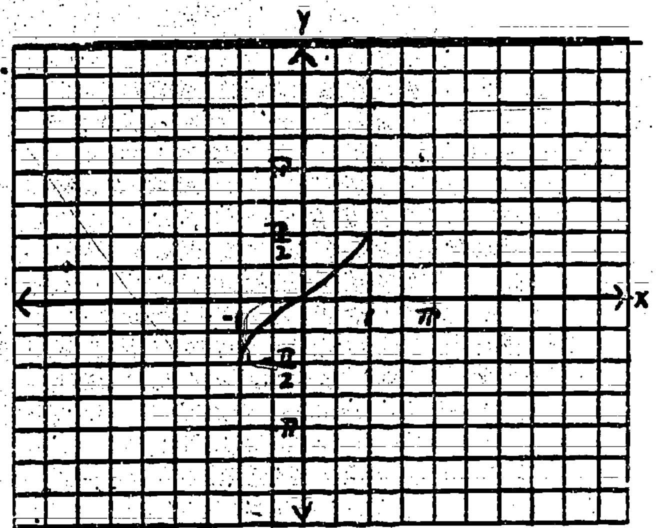
$-\cos 2\theta = 2 \sin^2 \theta - 1$

$\therefore \frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2 \sin^2 \theta - 1$

ANSWERS

XII-29

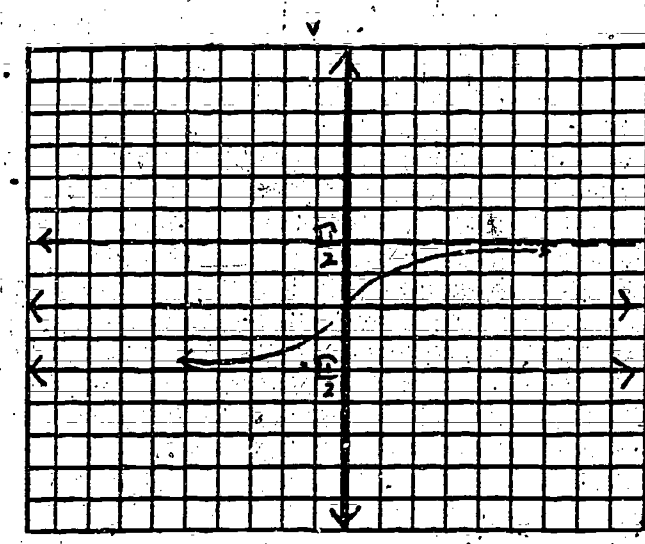
1.



Domain $-1 \le x \le 1$

Range $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

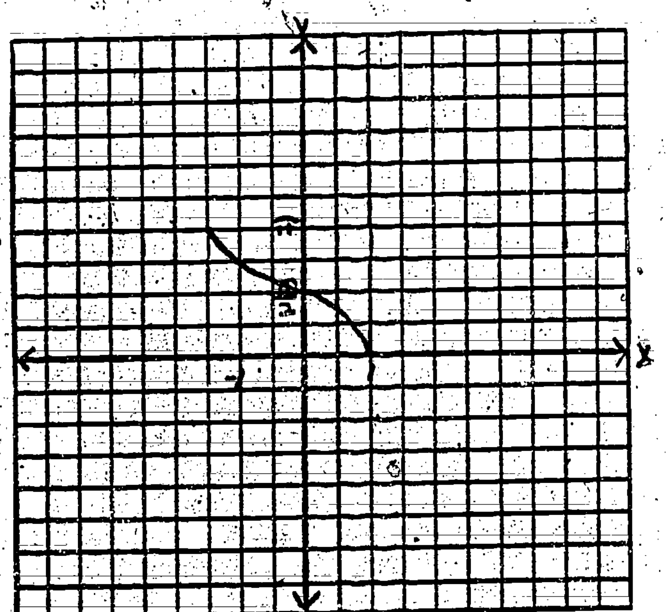
3.



Domain R

Range $-\frac{\pi}{2} < y < \frac{\pi}{2}$

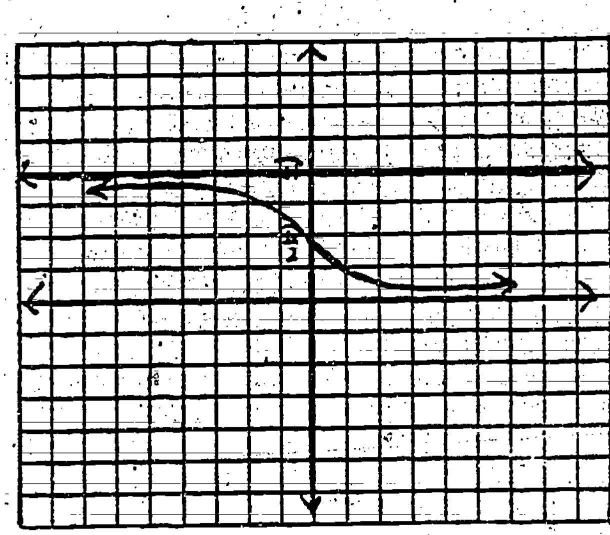
2.



Domain = $-1 \le x \le 1$

Range $\{y: 0 \le y \le \pi\}$

4.



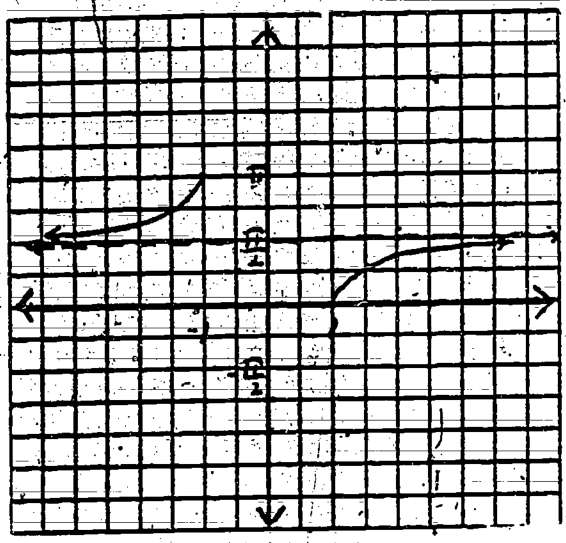
Domain = R

Range $\{y: 0 < y < \pi\}$

ANSWERS

XII-29 (continued)

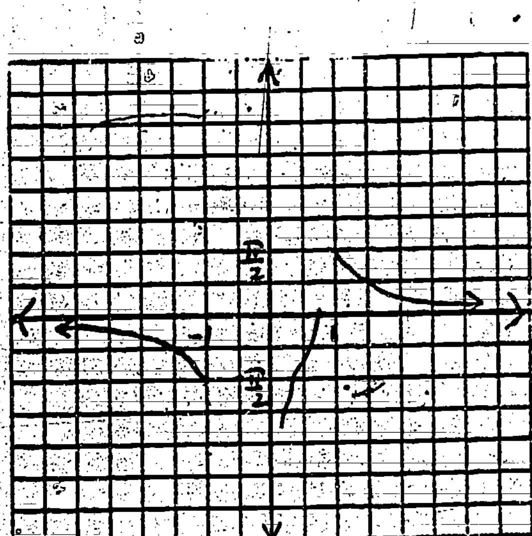
5.



Domain $\{x: |x| \geq 1\}$

Range $\{y: 0 \leq y \leq \pi,\ y \neq \frac{\pi}{2}\}$

6.



Domain $\{x: |x| \geq 1\}$

Range $\{y: -\frac{\pi}{2} \leq y \leq \frac{\pi}{2},\ y \neq 0\}$

XII-30

1. $-30°$
2. $19°$
3. $\frac{3}{5}$
4. $0°$
5. $\frac{\sqrt{2}}{2}$

XII-31

1. $\{x: x = \frac{\pi}{6} + 2k\pi \quad x = \frac{5}{6} + 2k$
2. $\{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}\}$
3. $\{\frac{\pi}{6}, \frac{5\pi}{6}\}$
4. General solution:
   $45° + 360k,\ 135° + 360k,$
   $225° + 360k,\ 315° + 360k$

   $\{45° + 180k, 135° + 180k\} \cup$
   $\{60° + 360k, 240° + 360k,$
   $120° + 360k, 300° + 360k\}$

   Particular Solution:

   $\{45, 60, 120, 135, 225, 240, 300, 315\}$

HIGHER ORDER ASSESSMENT

$22.5°$

ANSWERS

XII-32

1. 13.76 in.
2. 153.2 km
3. 46°
4. 10.36 feet
5. $151^\circ 30'$, $28^\circ 30'$
6. 154.7 in^2

XII-33

1. 7
2. 15
3. 137°
4. 12.3

XII-34

1. b = 537
2. a = 3037
3. a = 0.3466
4. a = 10.74

   c = 8.96

   m ∡ B = 98° 50'

XII-35

1. Two triangles exist

   m ∡ B = 47° 32' m ∡ C = 97° 28' c = 24.2

   m ∡ B' - 132° 28' m ∡ C = 12° 32' c' = 5.28

2. No triangle
3. One triangle

   a = 14.76, m A = 79° 40', m B = 69°
4. No triangle

XII-36

1. 15
2. 10
3. 114
4. 7°
5. 970.8 cm^2

XII-37

1. Law of Cosines
2. Law of Sines
3. Law of Sines
4. Law of Cosines

ANSWERS

ENRICHMENT 1

1. $(2\sqrt{2}, 45^\circ)$

2. $(1, 120^\circ)$

ENRICHMENT 2

1. $(\frac{5}{2}, \frac{5\sqrt{3}}{2})$

2. $(3, -3)$

ENRICHMENT 3

1. $x^2 - 4y + y^2 = 0$ or $x^2 + (y-2)^2 = 4$

2. $x^2(x^2 + y^2) - 4y^2 = 0$

ENRICHMENT 4

1. $r = \dfrac{2}{3 \cos \theta + \sin \theta}$

2. $r^2 + 2r \cos \theta = 0$ or $r = -2 \cos \theta$

ENRICHMENT 5

(See next page)

ENRICHMENT 6

1. $\sqrt{13} (\cos 33^\circ 48' + i \sin 33^\circ 48')$

2. $2 (\cos 330^\circ + i \sin 330^\circ)$

ENRICHMENT 7

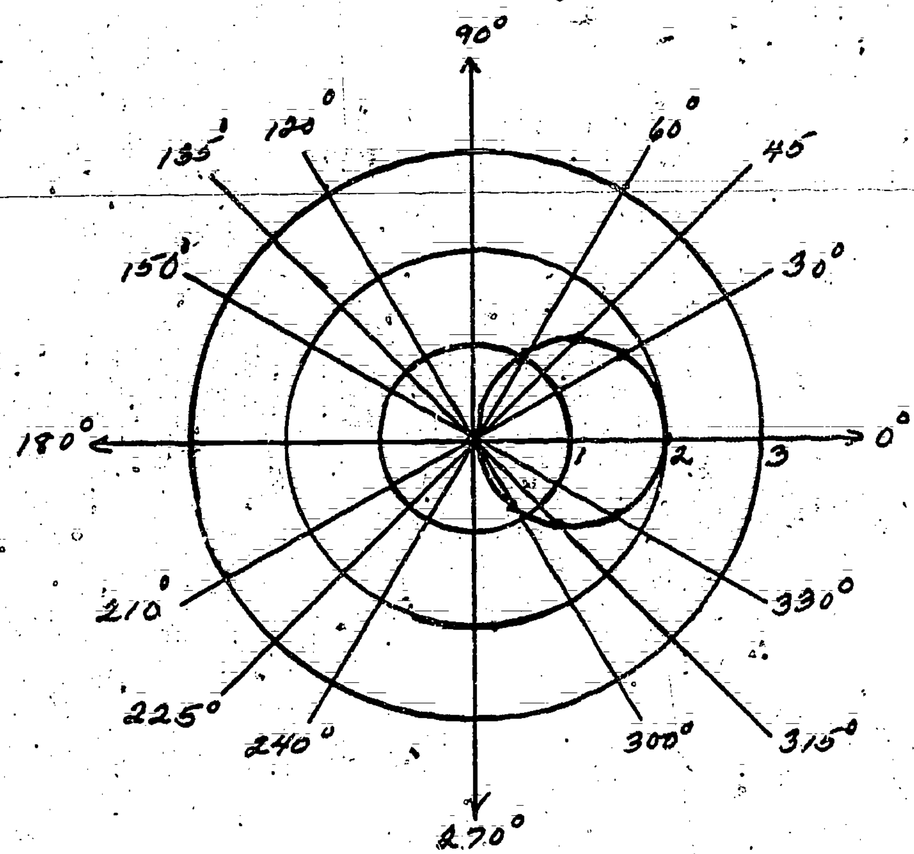
1. $6 (\cos 300^\circ + i \sin 300^\circ)$

2. $\frac{4}{5} (\cos 30^\circ + i \sin 30^\circ)$

ANSWERS

ENRICHMENT 5

1.



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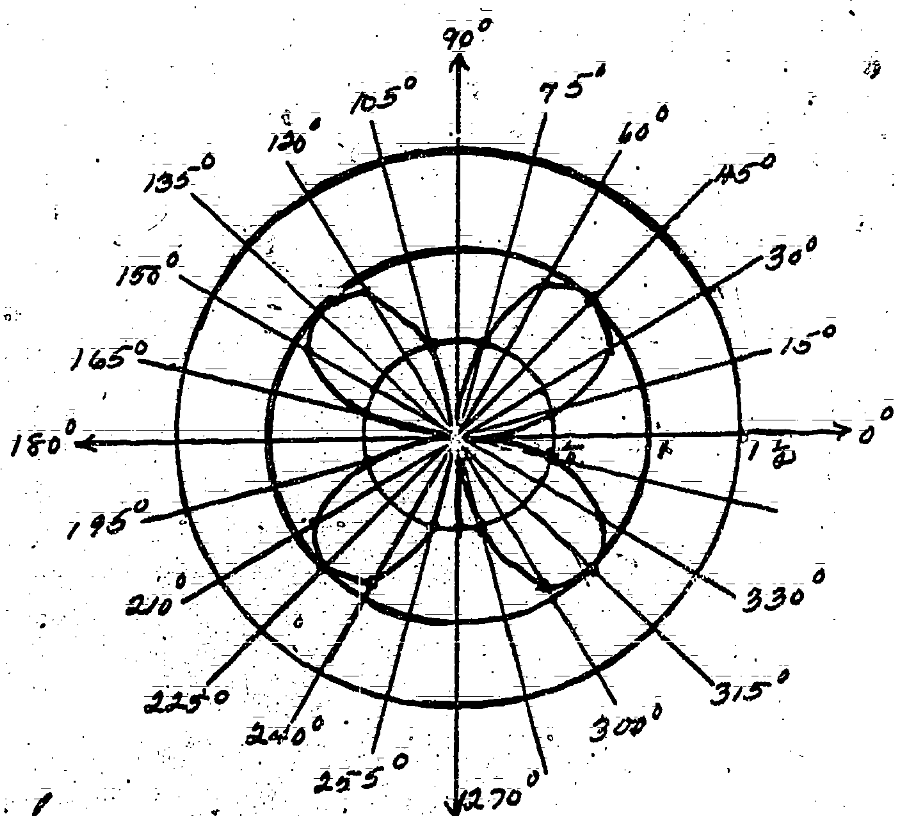

ANSWERS

ENRICHMENT 8

1. $32\ (\cos 150^\circ + i \sin 150^\circ)$ or $-16\sqrt{3} + 16i$

2. $\{3\ (\cos 30^\circ + i \sin 30^\circ),\ 3\ (\cos 150^\circ + i \sin 150^\circ),$
   $3\ (\cos 270^\circ + i \sin 270^\circ)\}$

or

$\{\frac{3\sqrt{3}}{2} + \frac{3}{2} i,\ -\frac{3\sqrt{3}}{2} + \frac{3}{2} i,\ -3i\}$

3. $\{2\ (\cos 0^\circ + i \sin 0^\circ),\ 2\ (\cos 90^\circ + i \sin 90^\circ),$
   $2\ (\cos 180^\circ + i \sin 180^\circ),\ 2\ (\cos 270^\circ + i \sin 270^\circ)\}$

or

$\{2, -2, 2i, -2i\}$

UNIT XIII - SEQUENCES, SERIES, AND THE BINOMIAL THEOREM

PURPOSE

Sequences, series, and the binomial theorem play a vital role in the mathematical interpretation of physical economics, and sociological situations and are appropriate to the development of students' mathematical knowledge at this time. Because of their importance in later mathematical studies, they are included in Algebra 2.

OVERVIEW

The unit develops the concepts and properties of sequences and series. These concepts are applied to problem-solving situations. The Binomial Theorem is used to expand binomials and to identify specific terms in an expression.

SUGGESTIONS TO THE TEACHER

The terms "progression" and "sequence" are synonymous terms in the unit. Some objectives have eight assessment tasks since they deal with both arithmetic and geometric sequences (series). The expansion mentioned in Objective 17 can be done using Pascal's triangle and/or the Binomial Theorem.

The following formulas are used in the unit:

a) $t_n = a + (n - 1)d$

b) $t_n = ar^{n-1}$

c) $S_n = \frac{n}{2}[2a + (n - 1)d]$

d) $S_n = \frac{n}{2}(a + a_n)$

e) $S_n = \frac{a - ar^n}{1 - r}, r \neq 1$

f) $S_n = \frac{a - ra_n}{1 - r}, r \neq 1$

g) $S = \frac{a}{1 - r}, r \neq 1$

h) r th term of $(a + b)^n = \frac{n(n - 1)(n-2) \ldots (n - r + 2)}{(r - 1)!} a^{n - r + 1} b, r > 1$

Time allocation for this unit is six days.

UNIT XIII - SEQUENCES, SERIES, AND THE BINOMIAL THEOREM

Computer Applications: BASIC BASIC, Coan, pp. 204-205; Algebra 2 and Trigonometry, Dolciani (1978), pp. 229, 243; Algebra 2 and Trigonometry, Dolciani, (1980), p. 226; Algebra Two with Trigonometry, Foster, p. 527; Computer Programming in the BASIC Language, Golden, pp. 170-171; Algebra Two and Trigonometry, Keedy, p. 540; Algebra Two with Trigonometry, Payne, pp. 524-526.

VOCABULARY

binomial expansion
sequence
progression
series
factorial
n^{th} (r^{th}) term

summation
sigma notation
first term: a
common difference: d
common ratio: r
last term: a_n
infinite series (finite)

UNIT XIII - SEQUENCES, SERIES, AND THE BINOMIAL THEOREM

PERFORMANCE OBJECTIVES

1. Identify a given sequence as arithmetic or geometric.
2. Write the first n terms of an arithmetic or geometric sequence.
3. Determine the n-th term of an arithmetic or geometric sequence.
4. Write the algebraic rule for a given arithmetic or geometric sequence.
5. Given three variables from an arithmetic or geometric sequence, determine the value of the fourth variable.
6. Determine a specified number of arithmetic or geometric means between two given elements of a sequence.
7. Write the expanded form of an arithmetic or geometric series, given the expression in summation notation.
8. Write, in summation notation, a given arithmetic or geometric series.
9. Determine the solution set for a linear equation involving summation notation.
10. Determine the sum of an arithmetic series.
11. Determine the sum of a finite geometric series.
12. Determine the sum of an infinite geometric series.
13. Identify the solution of a narrative problem involving an arithmetic sequence or series.
14. Determine the solution of a verbal problem involving a geometric sequence or series.
15. Demonstrate the procedure for finding an equivalent fraction for a given repeating decimal by using infinite geometric series.
16. State the simplified form for an expression involving factorial notation.
17. Write the expansion of $(a \pm b)^n$ for $n > 2$, $n \in$ natural number.
18. Determine the r-th term of the expansion of a binomial.

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OBJECTIVE	Dolciani (1978)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
1	507	217 230 233	213-216 226-230	360-362 369-371	524 531	395	505 510	554	--
2	495-497 506-507	219 233	213-216 226-230	360-362 369-371	526 533	396-400	505 513	543-545 553-555	389-391 401-403
3	495-498 506-508	222 233	213-216 226-230	360-362 369-371	526 533	396-400	505, 508 510, 513	543-545 553-555	389-392 401-403
4	495-498 506-508	219 233	213-216 226-230	360-362 369-371	521	393	--	543-545 553-555	389-392 401-403
5	495-497 506-508	220 233	213-216 226-230	360-362 369-371	526-527	396 400	508 510	543-545 553-556	389-392 401-403
6	498-500 509-511	220-223 235-238	216-220 231-235	360-362 369-371	--	396 400	511	546-548 553-555	393-396 404-408
7	501-503 511	225-227 240-241	220-225 235-239	364-374	523	--	--	549-550 557-558	393-396 404-408
8	501-503	225-227 240-241	220-225 235-239	366-368 372-374	523	--	--	549-551	393-396 401-405
9	504	--	--	364-365	--	--	--	551-552	--
10	501-503	224-227	220-225	366-368	528-530	401-405	506-508	549-551	392-396

CROSS REFERENCE TO CURRENTLY USED AND/OR APPROVED TEXTS

OVJECTIVE	Dolciani (1973)	Dolciani (1978)	Dolciani (1980)	Foster (1979)	Keedy (1978)	Payne (1977)	Sobel (1977)	Sorgenfrey (1973)	Travers (1978)
11	511-513	240-241	235-239	372-374	534-535	406-407	511-513	557-559	404-406
12	515-516	249-252	244-250	375-378	536-539	408-410	514-516	560-562	411-413
13	397-398 500	223-224 228	219-220 224-225	371	527-530	397 405	509	545-546 548	397-399
14	508-509	234-238 242	230-231 234-235 238-239	378	533-539	401 407	514	556	407-409
15	515-516	253	248-248	375-378	539	408 410	517	560-562	411-413
16	519-520	410	425	385-387	569-570	419 420	525-526	565-567	420-421
17	595-596 519-521	420-423	436-438	385-387	579-581	89-93	521-524	585-586 563-567	414-417 420-422
18	519-521	422	436-438	385-387	581	92-93	529	563-567	414-417 420-422

XIII-5

PERFORMANCE OBJECTIVE XIII-1

Identify a given sequence as arithmetic or geometric.

1. State whether the following sequences are arithmetic, geometric, both, or neither.

   a) $\{2, 4, 6, 8, \ldots\}$.

   b) $\{\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots\}$.

   c) $\{3, 4, 5, 6, \ldots\}$.

   d) $\{2, 2, 2, 2, \ldots\}$.

   $\{3, -3, 3, -3, \ldots\}$.

2. State whether the following sequences are arithmetic, geometric, both, or neither.

   a) $\{a, 2a, 4a, 6a, \ldots\}$.

   b) $\{a\sqrt{2}, 3a\sqrt{2}, 5a\sqrt{2}, 7a\sqrt{2}, \ldots\}$.

   c) $\{1, 2, 3, 5, 8, \ldots\}$.

   d) $\{2, 4, 8, 16, 32, \ldots\}$.

   e) $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \ldots\}$.

3. State whether the following sequences are arithmetic, geometric, both, or neither.

   a) $\{a, 2a, 3a, 4a, 5a, \ldots\}$.

   b) $\{2, 5, 8, 11, 14, \ldots\}$.

   c) $\{1, 3, 5, 7, 9, \ldots\}$.

   d) $\{1, 4, 9, 16, 25, \ldots\}$.

   e) $\{2, 4, 6, 8, 10, \ldots\}$.

4. State whether the following sequences are arithmetic, geometric, both or neither.

   a) $\{3, 6, 12, 24, \ldots\}$.

   b) $\{-2a^2, 0, 2a^2, 4a^2, \ldots\}$.

   c) $\{-19, -15, -13, -12, \ldots\}$.

   d) $\{-1, -2, -4, -8, \ldots\}$.

   e) $\{-15, -5, 4, 12, 19\}$.

PERFORMANCE OBJECTIVE XIII-2

Write the first n terms of an arithmetic or geometric sequence.

1. Write the next three terms of the arithmetic sequence {2, 6, 10, ...} ...}.
2. Write the next three terms of the arithmetic sequence {4, 7, 10, 13, ...}.
3. Write the next three terms of the arithmetic sequence {-1, -3, -5, ...}.
4. Write the next three terms of the arithmetic sequence {-2, -10, -18, ...}.
5. Write the first four terms of a geometric sequence if $a = -3$ and $r = -\frac{1}{2}$.
6. Write the first four terms of a geometric progression if $a = 2$ and $r = \frac{1}{3}$.
7. Write the first four terms of a geometric progression if $a = 2x$ and $r = 3$.
8. Write the first four terms of a geometric progression if $a = 4y + 3$ and $r = \frac{1}{2}$.

PERFORMANCE OBJECTIVE XIII-3

Determine the n-th term of an arithmetic or geometric sequence.

1. Determine the 5th term of a geometric sequence whose first term is 6 and common ratio is $\frac{1}{3}$.
2. Determine the 4th term of a geometric sequence whose first term is -4 and common ratio is 4.
3. Determine the 6th term of a geometric sequence whose first term is $3\sqrt{5}$ and common ratio is 2.
4. Determine the 25th term of the geometric sequence: $\{1, -1, 1, -1, \ldots\}$
5. Determine the 5th term of an arithmetic sequence having $a = 2$ and $d = 4$.
6. Determine the 19th term of the arithmetic sequence $\{-8, -2, 4, 10, \ldots\}$.
7. Determine the 20th term of an arithmetic sequence $\{6, 3, 0, -3, \ldots\}$.
8. A man has been employed 17 years and had a starting salary of $8600 with a yearly increase of $450. The man's salary during the 17th year of employment is ____.

PERFORMANCE OBJECTIVE XIII-4

Write the algebraic rule for a given arithmetic or geometric sequence.

1. Write the algebraic rule for the arithmetic sequence $\{3, 5, 7, 9, \ldots\}$.

2. Write the algebraic rule for the arithmetic sequence $\{22, 15, 8, 1 \ \ldots\}$.

3. Write the algebraic rule for the arithmetic sequence $\{-13, -5, 3, 11, \ldots\}$.

4. Write the algebraic rule for the arithmetic sequence $\{2x, 2x + 3, 2x + 6, 2x + 9, \ldots\}$.

5. Write the algebraic rule for the geometric progression $\{2, 1, \frac{1}{2}, \frac{1}{4}, \ldots\}$.

6. Write the algebraic rule for the geometric progression $\{-3, -1, -\frac{1}{3}, -\frac{1}{9}, \ldots\}$.

7. Write the algebraic rule for the geometric progression $\{4, -\frac{1}{2}, \frac{1}{16}, -\frac{1}{28}, \ldots\}$,

8. Write the algebraic rule for the geometric progression $\{-5, \frac{5}{3}, -\frac{5}{9}, \frac{-5}{27} \ \ldots\}$.

PERFORMANCE OBJECTIVE XIII-5

Given three variables from an arithmetic or geometric sequence, determine the value of the fourth variable.

1. Given an arithmetic sequence where $a = -2$, $n = 5$, $d = -5$, determine t_5.

2. Given an arithmetic progression where $t_{21} = 100$ and $d = 2$, determine a.

3. Given an arithmetic progression where $t_{16} = 110$, $a = 65$, and $n = 16$, determine d.

4. Given an arithmetic progression where $t_n = -122$, $a = -22$, $d = -20$, determine n.

5. Given a geometric sequence where $a = -2$, $r = 2$, and $n = 5$, determine t_5.

6. Given a geometric sequence where $t_3 = 10$, $r = \frac{1}{5}$, $n = 3$, determine a.

7. Given a geometric sequence where $t_3 = 625$, $a = 25$, $n = 3$, determine two possible values of r.

8. Given a geometric sequence where $t_n = 81$, $a = 3$, $r = 3$, determine n.

PERFORMANCE OBJECTIVE XIII-6

Determine a specified number of arithmetic or geometric means between two given elements of a sequence.

1. Determine three geometric means between -7 and -112.

2. Determine the mean proportional between 6.4 and 10.

3. Determine four geometric means between $\frac{2}{7}$ and 4802.

4. Determine two geometric means between m^2 and m^{10}.

5. Determine two arithmetic means between 24 and 32.

6. Determine two arithmetic means between 6 and 12.

7. Determine three arithmetic means between 18 and 24.

8. Determine two arithmetic means between 3 and 921.

PERFORMANCE OBJECTIVE XIII-7

Write the expanded form of an arithmetic or geometric series, given the expression in summation notation.

1. Write the geometric series represented by $\sum_{K=1}^{3} 5(3)^{K-1}$.

2. Write the geometric series represented by $\sum_{j=2}^{5} -2\ (^{-}1)^{j-2}$.

3. Write the geometric series represented by $\sum_{K=1}^{3} (4)\ (-3)^{K-1}$.

4. Write the geometric series represented by $\sum_{i=1}^{3} -\frac{1}{2}\ (-\frac{1}{3})^{i-1}$.

5. Write the arithmetic series represented by $\sum_{n=1}^{5} (3n - 1)$.

6. Write the arithmetic series represented by $\sum_{j=5}^{11} (2j + 1)$.

7. Write the arithmetic series represented by $\sum_{K=6}^{9} \frac{1}{2}\ (K + \frac{1}{2})$.

8. Write the arithmetic series represented by $\sum_{M=20}^{25} (8 - 2M)$.

PERFORMANCE OBJECTIVE XIII-8

Write, in summation notation, a given arithmetic or geometric series.

1. Use sigma notation to write the sum of the geometric series

   $2 + 4 + 8 + 16 + 32.$

2. Use sigma notation to write the sum of the geometric series

   $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}.$

3. Use sigma notation to write the sum of the geometric series

   $\frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64}.$

4. Use sigma notation to write the sum of the geometric series

   $(-8) + (-2) + (-\frac{1}{2}) + (-\frac{1}{8}).$

5. Write, in summation notation, the arithmetic series $2 + 3 + 4 + 5.$

6. Write, in summation notation, the arithmetic series $5 + 7 + 9 + 11 + 13.$

7. Write, in summation notation, the arithmetic series

   $1 + 0 + (-1) + (-2) + (-3).$

8. Write, in summation notation, the arithmetic series

   $1 + (-1) + (-3) + (-5).$

PERFORMANCE OBJECTIVE XIII-9

Determine the solution set for a linear equation involving summation notation.

1. Determine the solution set of the equation

$$\sum_{a=4}^{6} (ab + 1) = 33 .$$

2. Determine the solution set of the equation

$$\sum_{b=3}^{6} (ab + 1) = 32 .$$

3. Determine the solution set of the equation

$$\sum_{y=5}^{8} (2xy - 3) = -12 .$$

4. Determine the solution set of the equation

$$\sum_{x=1}^{3} (-2x + y) = 288 .$$

PERFORMANCE OBJECTIVE XIII-10

Determine the sum of an arithmetic series.

1. Determine the sum of the arithmetic series

$$\sum_{n=1}^{10} (2n)$$

2. Determine the sum of the arithmetic series

$$\sum_{a=1}^{16} (a + 2)$$

3. Determine the sum of the arithmetic series

$$\sum_{a=1}^{21} (2a + 1)$$

4. Determine the sum of the arithmetic series

$$\sum_{x=1}^{20} (2 - 3x)$$

PERFORMANCE OBJECTIVE XIII-11

Determine the sum of a finite geometric series.

1. Determine the sum of the geometric series

$$\sum_{a=1}^{5} 4\ (3)^{a-1}.$$

2. Determine the sum of the geometric series

$$\sum_{x=1}^{6} (2\ (4)^{x-1}.$$

3. Determine the sum of the geometric series

$$\sum_{x=2}^{6} 10\ (4)^{x-2}.$$

4. Determine the sum of the geometric series

$$\sum_{K=1}^{5} \left(-\frac{1}{3}\right) \left(-\frac{1}{2}\right)^{K-1}.$$

PERFORMANCE OBJECTIVE XIII-12

Determine the sum of an infinite geometric series.

1. Determine the sum of the infinite geometric series whose first term is 6 and common ratio is $\frac{1}{3}$.

2. Determine the sum of the infinite geometric series $10, -5, \frac{5}{2}, -\frac{5}{4}, \ldots$

3. Determine the sum of the infinite geometric series

$$\sum_{M=1}^{\infty} \left(\frac{1}{10}\right)^{n}.$$

4. Determine the sum of the infinite geometric series whose first term is 51 and common ratio is .25.

Identify the solution of a narrative problem involving an arithmetic sequence or series.

1. A taxi rate is 80¢ for the first mile and 50¢ per mile for each additional mile. The fare for nine miles of travel is:

   a) \$4.60
   b) \$4.30
   c) \$5.30
   d) \$4.80
   e) \$7.70

2. An auditorium is designed with 10 seats in the first row and three seats more in each succeeding row. The number of seats in the first 20 rows is:

   a) 770
   b) 280
   c) 870
   d) 320
   e) None of the above

3. A child's weekly allowance increased yearly in an arithmetic sequence. If his allowance was \$1.20 per week the first year and was \$4.45 per week the sixth year, then his allowance the third year was:

   a) \$1.80
   b) \$2.50
   c) \$4.45
   d) \$4.00
   e) \$2.75

4. The sum of the positive integers less than 50 that are divisible by 3 is:

   a) 388
   b) 368
   c) 348
   d) 428
   e) 408

PERFORMANCE OBJECTIVE XIII-14

Determine the solution of a verbal problem involving a geometric sequence or series.

1. A father gives his son \$1.00 on his tenth birthday and doubles it every year. Determine how much the father gives the son on his 20th birthday.

2. The value of a car depreciates 20% the first year and 10% each year after that. Determine the value of a 4-year old car which originally sold for \$4,000. (Compute the answer to the nearest dollar.)

3. A city council committee on planning estimates the city's population will increase 5% annually. If the present population is 5,000, determine the projected population at the end of five years.

4. A ball dropped 50 meters above a hard surface rebounds on each bounce $\frac{4}{5}$ of the distance from which it fell. Determine the number of meters the ball will travel if it lands in the mud after the fourth bounce.

PERFORMANCE OBJECTIVE XIII-15

Demonstrate the procedure for finding an equivalent fraction for a given repeating decimal by using infinite geometric series.

1. Demonstrate the procedure for finding an equivalent common fraction for 0.777... in lowest terms.

2. Demonstrate the procedure for finding an equivalent common fraction for $0.1\overline{2}$, ... in lowest terms.

3. Demonstrate the procedure for finding an equivalent common fraction for $0.6\overline{75}$, ... in lowest terms.

4. Demonstrate the procedure for finding an equivalent common fraction for $0.1\overline{29}$,... in lowest terms.

PERFORMANCE OBJECTIVE XIII-16

State the simplified form for an expression involving factorial notation.

1. State the simplified form (without factorials) of the expression

$(4!)(3!)$.

2. State the simplified form (without factorials) of the expression

$\frac{(6!)(3!)}{5!}$.

3. State the simplied form (without factorials) of the expression

$\frac{(r-1)}{r!}$, when $r = 4$.

4. State the simplified form (without factorials) of the expression

$\frac{(8)(7)(6)(5)(4)(3)(2)(5!)}{7!}$.

PERFORMANCE OBJECTIVE XIII-17

Write the expansion of $(a \pm b)^n$ for $n > 2$, $n \in$ natural numbers.

1. Write the expansion of $(a + b)^i$ where $n = 3$ by using Pascal's triangle.

2. Write the expansion of $(a + b)^i$ where $n = 4$ by using Pascal's triangle.

3. Write the expansion of $(a - b)^i$ where $n = 5$ by using Pascal's triangle.

4. Write the expansion of $(a - b)^i$ where $n = 6$ by using Pascal's triangle.

5. Write the expansion of $(x + 1)^4$.

6. Write the expansion of $(x + 2y)^5$.

7. Write the expansion of $(2x - 3y)^4$.

8. Write the expansion of $(-x^2 - 2y)^6$.

PERFORMANCE OBJECTIVE XIII-18

Determine the r-th terms of the expansion of a binomial.

1. Determine the third term of $(3x + y)^6$.

2. Determine the fourth term of $(a + 2b)^5$.

3. Determine the fifth term of $(4x - y)^7$.

4. Determine the middle term of $(y + \frac{y}{3})^8$.

ANSWERS

XIII-1

1. a) arithmetic
   b) geometric
   c) arithmetic
   d) both
   e) geometric
2. a) neither
   b) arithmetic
   c) geometric
   d) geometric
   e) neither
3. a) arithmetic
   b) arithmetic
   c) geometric
   d) arithmetic
   e) neither
4. a) geometric
   b) arithmetic
   c) neither
   d) geometric
   e) neither

XIII-2

1. {14, 18, 22}
2. {16, 19, 22}
3. {-7, -9, -11}
4. {-26, -34, -42}

XIII-2 (continued)

5. $\{-3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8}\}$
6. $\{2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}\}$
7. {2x, 6x, 18x, 54x}
8. $\{4y + 3,\ 2y + \frac{3}{2},\ y + \frac{3}{4},\ \frac{y}{2} + \frac{3}{8}\}$

XIII-3

1. $\frac{2}{27}$
2. -256
3. $96\sqrt{5}$
4. 1
5. 18
6. 100
7. -51
8. 15,800

XIII-4

1. $t_n = 3 + 2\ (n - 1)$
2. $t_n = 22 - 7\ (n - 1)$
3. $t_n = -13 + 8\ (n - 1)$
4. $t_n = 2x + 3\ (n - 1)$
5. $t_n = 2\ (\frac{1}{2})^{n-1}$
6. $t_n = (-3)\ (\frac{1}{3})^{n-1}$
7. $t_n = 4\ (-\frac{1}{8})^{n-1}$
8. $t_n = (-5)\ (-\frac{1}{3})^{n-1}$

ANSWERS

XIII-5

1. $t_n = -22$
2. $a = 60$
3. $d = 3$
4. $n = 6$
5. $t_n = -32$
6. $a = 250$
7. $r = \pm 5$
8. $n = 4$

XIII-6

1. $-14, -28, -56$
   OR
   $-14, -28, 56$
2. ± 8
3. $2, 14, 98, 686$
4. $M^{\frac{14}{3}}, M^{\frac{22}{3}}$
5. $\frac{80}{3}, \frac{68}{3}$
6. $8, 10$
7. $\frac{39}{2}, 21, \frac{45}{2}$
8. $156, 309, 462, 615, 768$

XIII-7

1. $5 + 15 + 45$
2. $(-2) + (2) + (-2) + (2)$
3. $4 - 12 + 36$
4. $(-\frac{1}{2}) + (\frac{1}{6}) + (-\frac{1}{18})$

XIII-7 (continued)

5. $2 + 5 + 8 + 11 + 14$
6. $11 + 13 + 15 + 17 + 19 + 21 + 23$
7. $3\frac{1}{4} + 3\frac{3}{4} + 4\frac{1}{4} + 4\frac{3}{4}$,or
   $\frac{13}{4} + \frac{15}{4} + \frac{17}{4} + \frac{19}{4}$
8. $(-32) + (-34) + (-36) +$
   $(-38) + (-40) + (-42)$

XIII-8

1. $\sum_{n=1}^{5} (2)^n$
2. $\sum_{n=1}^{4} (\frac{1}{3})^{n-1}$
3. $\sum_{n=1}^{5} (-\frac{1}{2})^{n+1}$
4. $\sum_{x=1} (-2)$
5. $\sum_{i=1}^{4} (i + 1)$
6. $\sum_{i=3}^{7} (2i - 1)$
7. $\sum_{K=1}^{4} (-2K + 3)$
8. $\sum_{K=1}^{4} (-2K + 3)$

ANSWERS

XIII-9

1. $b = 2$
2. $a = \frac{14}{9}$
3. $x = 0$
4. $y = 100$

XIII-10

1. 110
2. 168
3. 483
4. -590

XIII-11

1. 484
2. 2730
3. 3410
4. $-\frac{11}{48}$

XIII-12

1. 9
2. $6\frac{2}{3}$
3. $\frac{1}{9}$
4. 68

XIII-13

1. d)
2. a)
3. b)
4. a)

XIII-14

1. $1024
2. $2333
3. 6381
4. 286.16 meters

XIII-15

1. $\frac{7}{9}$
2. $\frac{11}{90}$
3. $\frac{223}{330}$
4. $\frac{64}{495}$

XIII-16

1. 144
2. 36
3. $\frac{1}{8}$
4. 960

XIII-17

1. $a^3 + 3a^2b + 3ab^2 + b^3$
2. $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
3. $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$
4. $a^6 - 6a^5b + 15a^4b^2 - 20a^2b^4 + 15a^2b^4 - 6ab^5 + b^6$

ANSWERS

XIII-17 (continued)

5. $(x + 1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$

6. $(x + 2y)^5 = x^5 + 10x^4 y + 40x^3y^2 + 80x^2y^2 + 80xy^4 + 35y^5$

7. $(2x - 3y)^4 = 16x^4 - 24x^3y + 36x^2y^2 - 54xy^3 - 54xy^3 + 81y^4$

8. $(-x - 2y)^6 = x^{12} + 12x^{10}y + 60x^8y^2 + 160x^8y^2 + 240x^4y^4 + 192x^2y^5 + 64y^6$

XIII-13

1. $1215x^4y^2$

2. $80a^2b^3$

3. $2240x^3y^4$

4. $\frac{70y^{12}}{81}$